Optimal Sampling from Distributed Streams

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Joint work with
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MSRA

Reservoir sampling [Waterman '??; Vitter '85]

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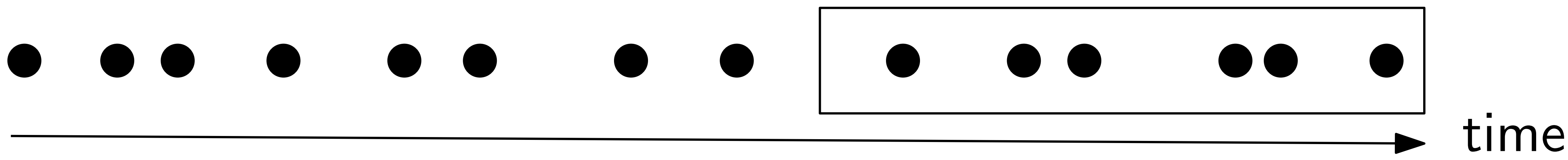
- **Correctness:** intuitive
- **Cost:** Space: $O(s)$, time $O(1)$

Sampling from a sliding window

[Babcock, Datar, Motwani, SODA'02; Gemulla, Lehner, SIGMOD'08; Braverman, Ostrovsky, Zaniolo, PODS'09]



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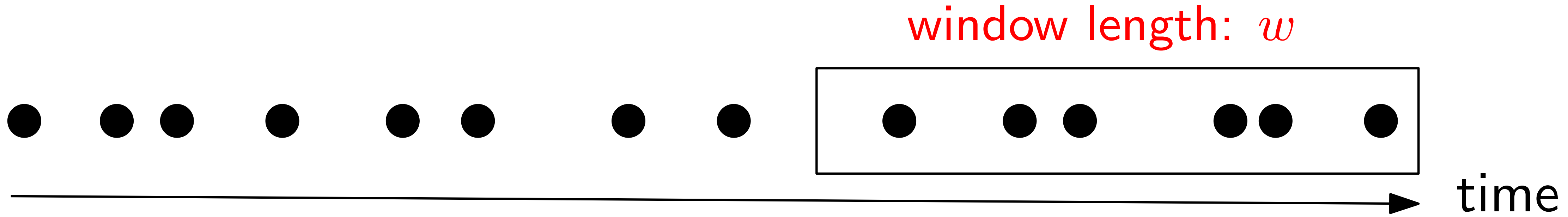

Time based window and sequence based window

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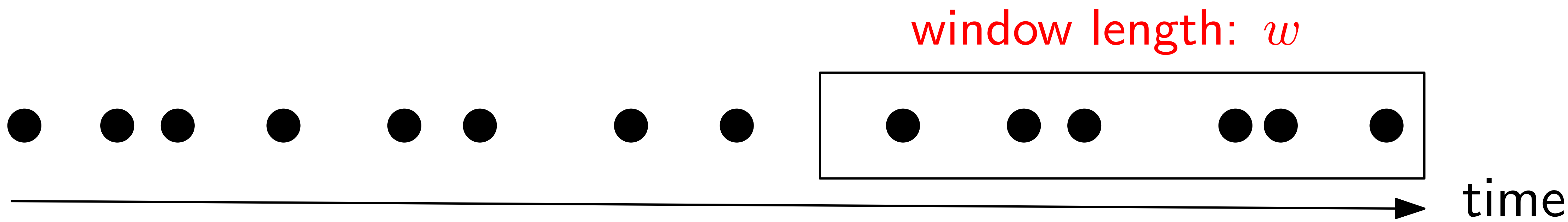
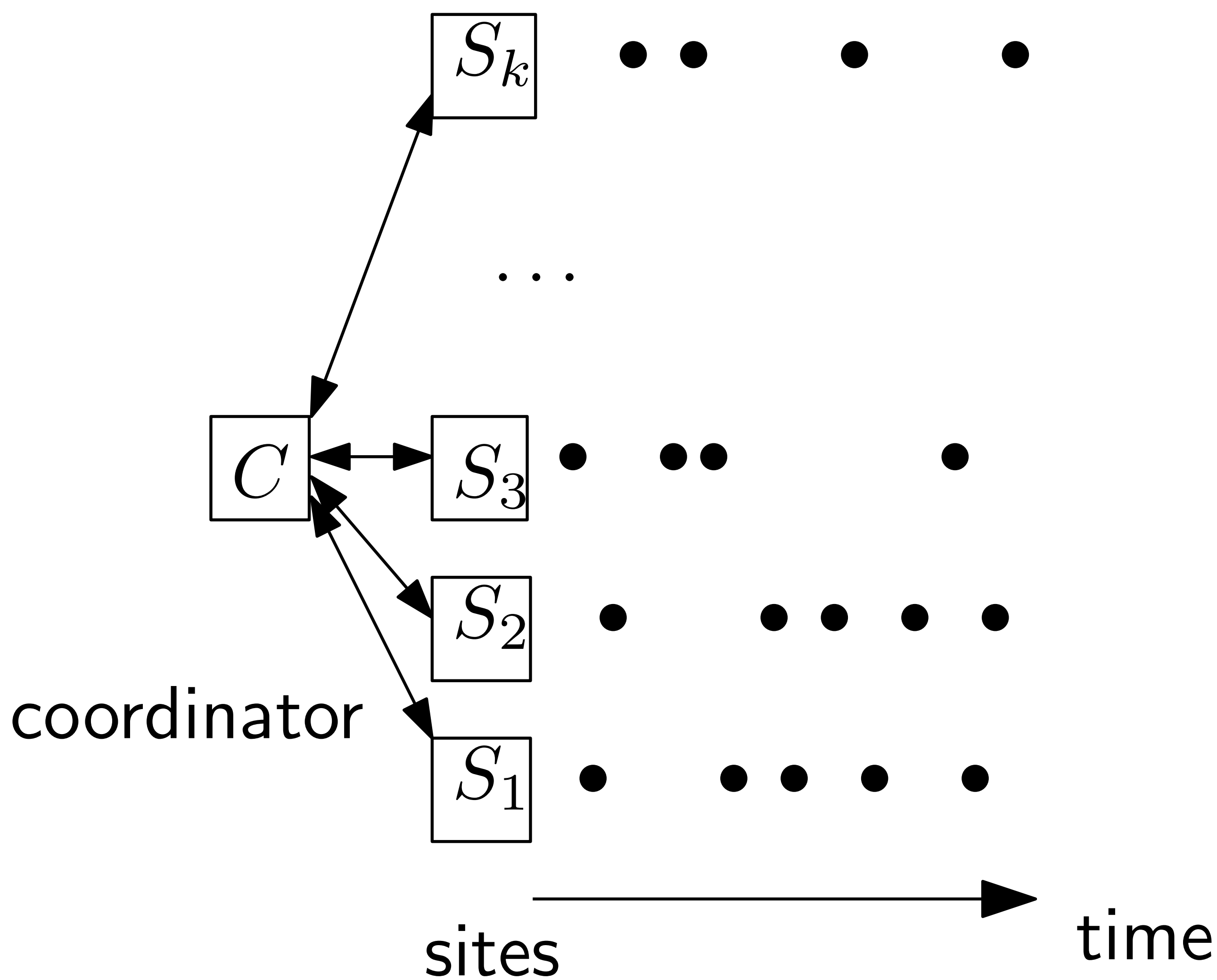


Time based window and sequence based window

- Space: $\Theta(s \log w)$
  - w: number of items in the sliding window
- Time: $\Theta(\log w)$

Sampling from distributed streams

- Maintain a (uniform) sample (w/o replacement) of size s from k streams of a total of n items



Primary goal:
communication

Secondary goal:
space/time at coordinator/site

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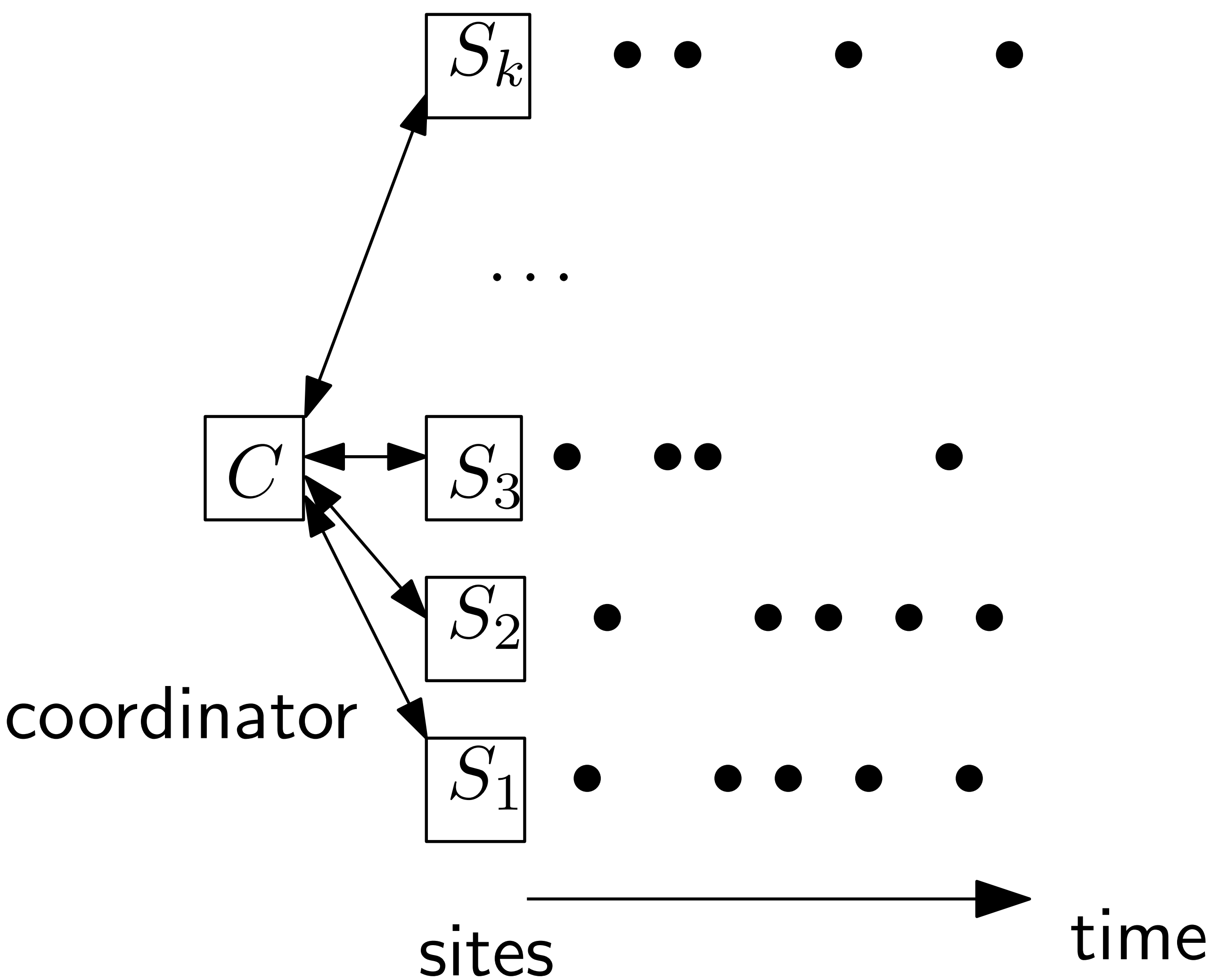

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Applications:

Internet routers
Sensor networks
Distributed computing

Why existing solutions don't work

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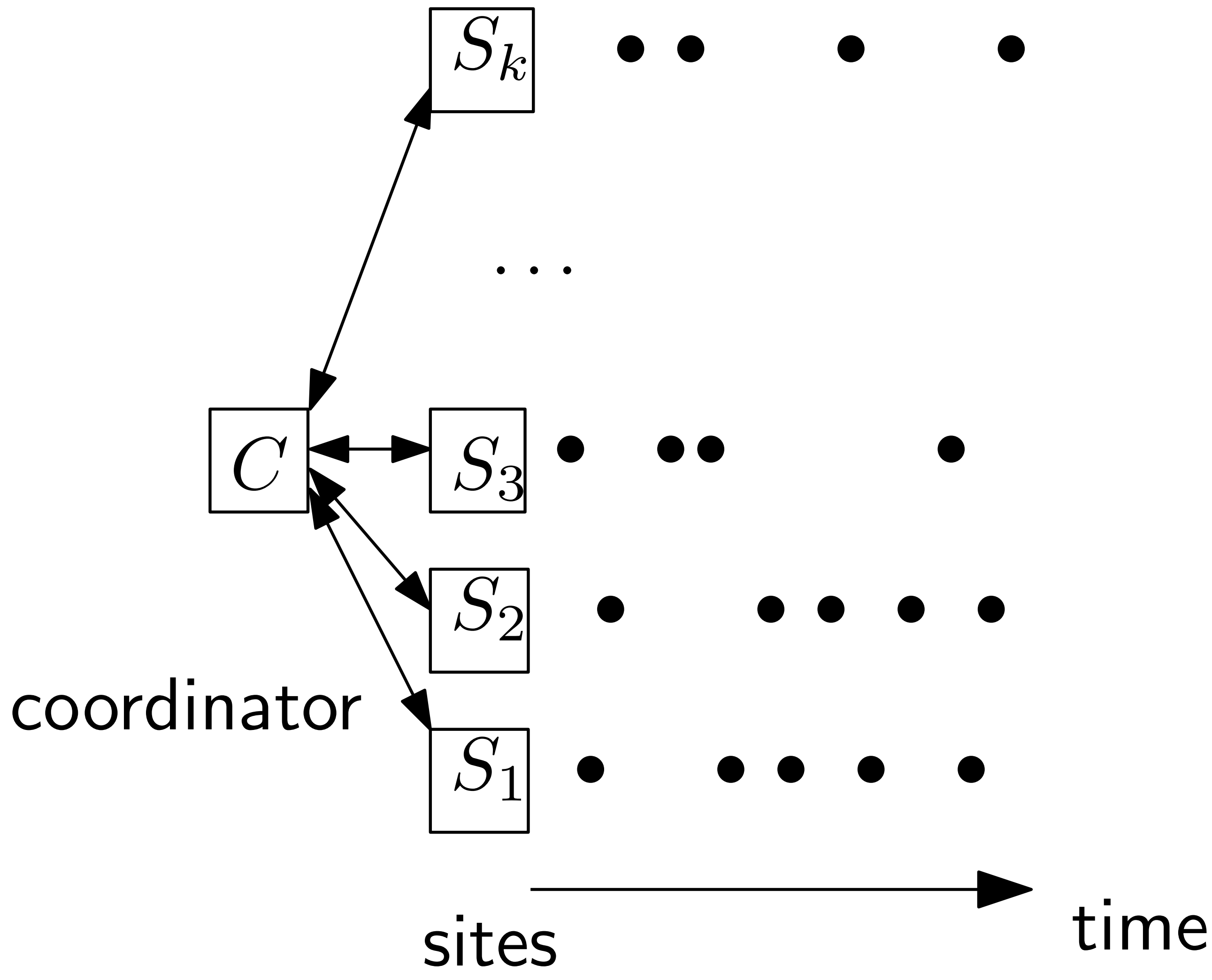

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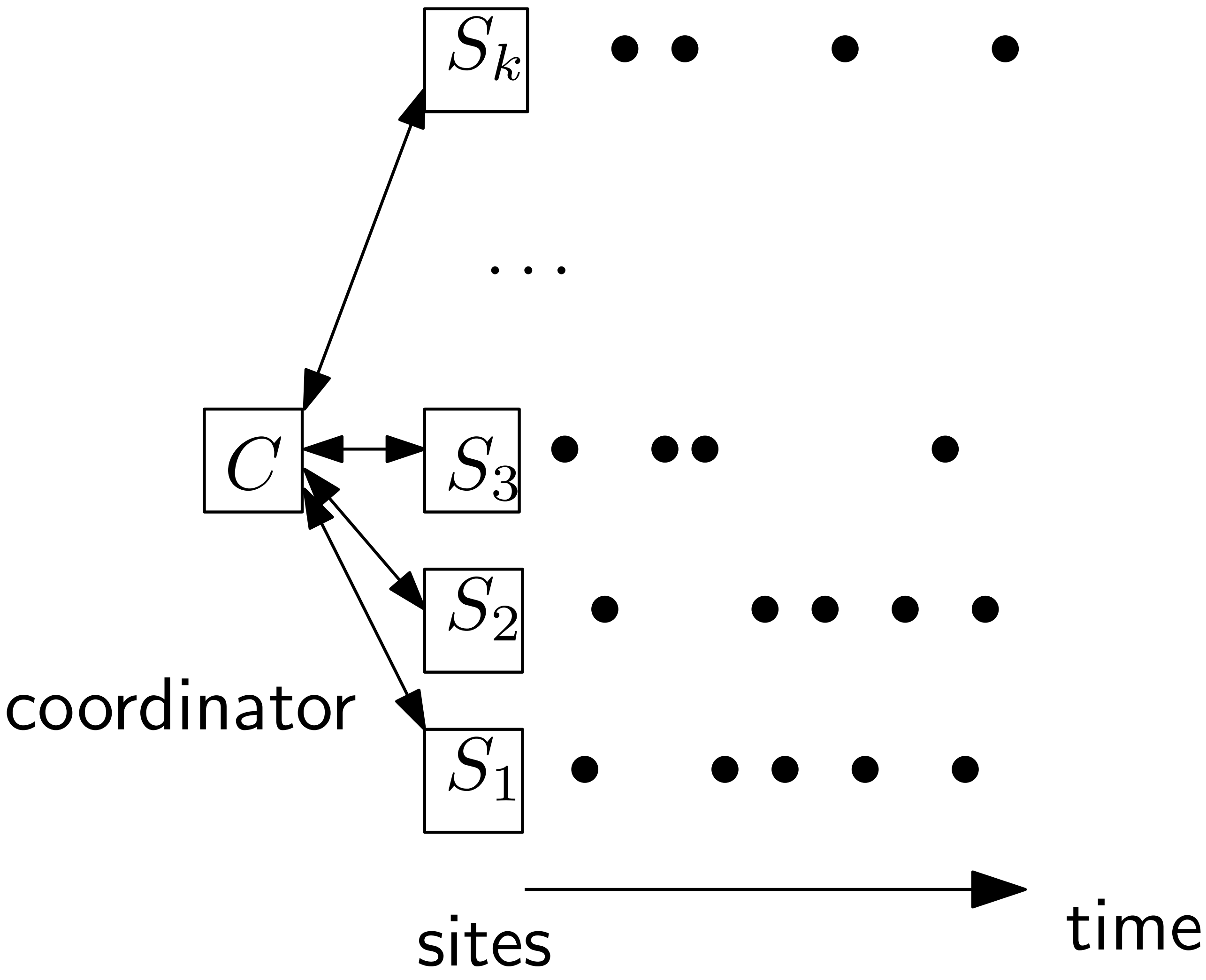

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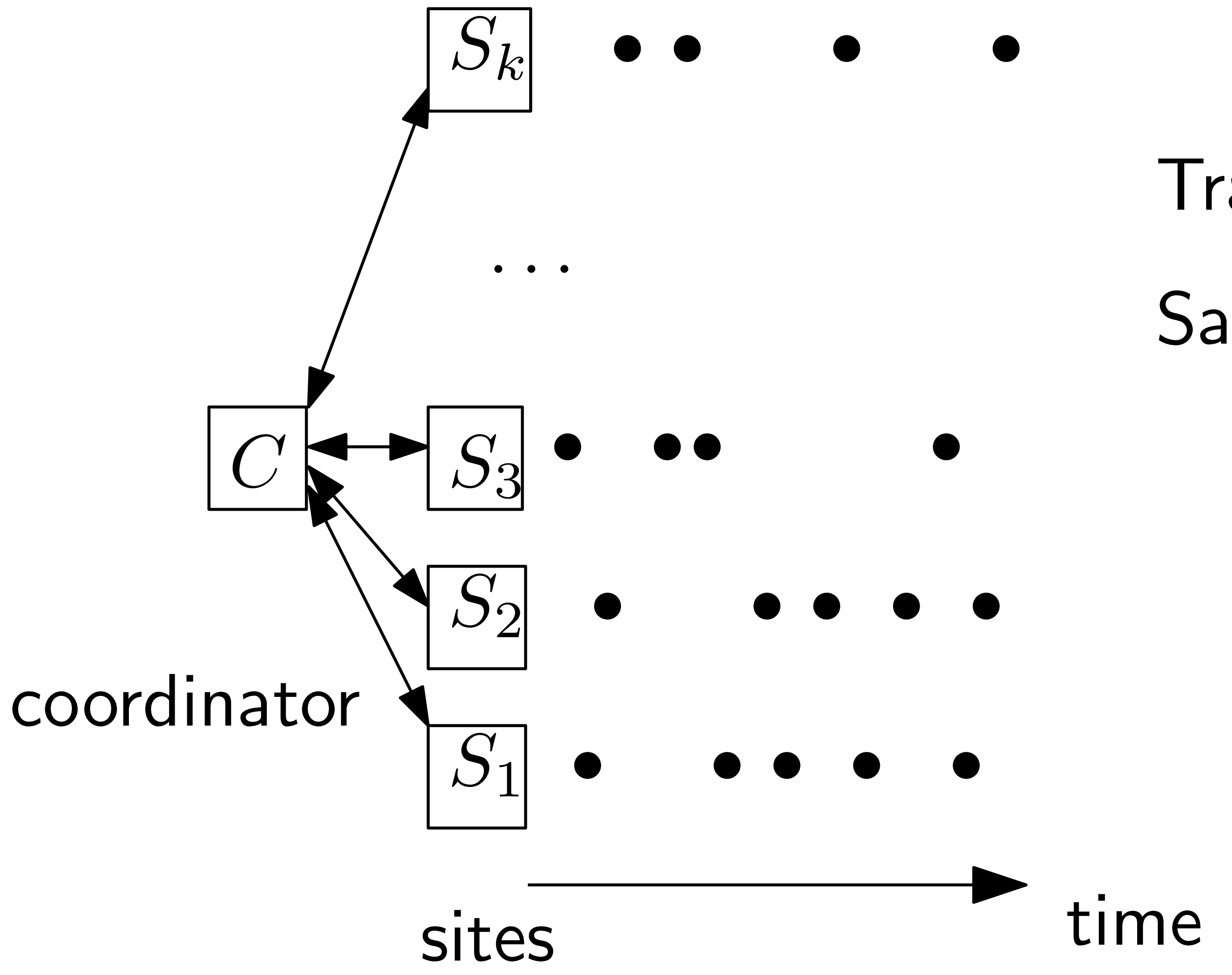

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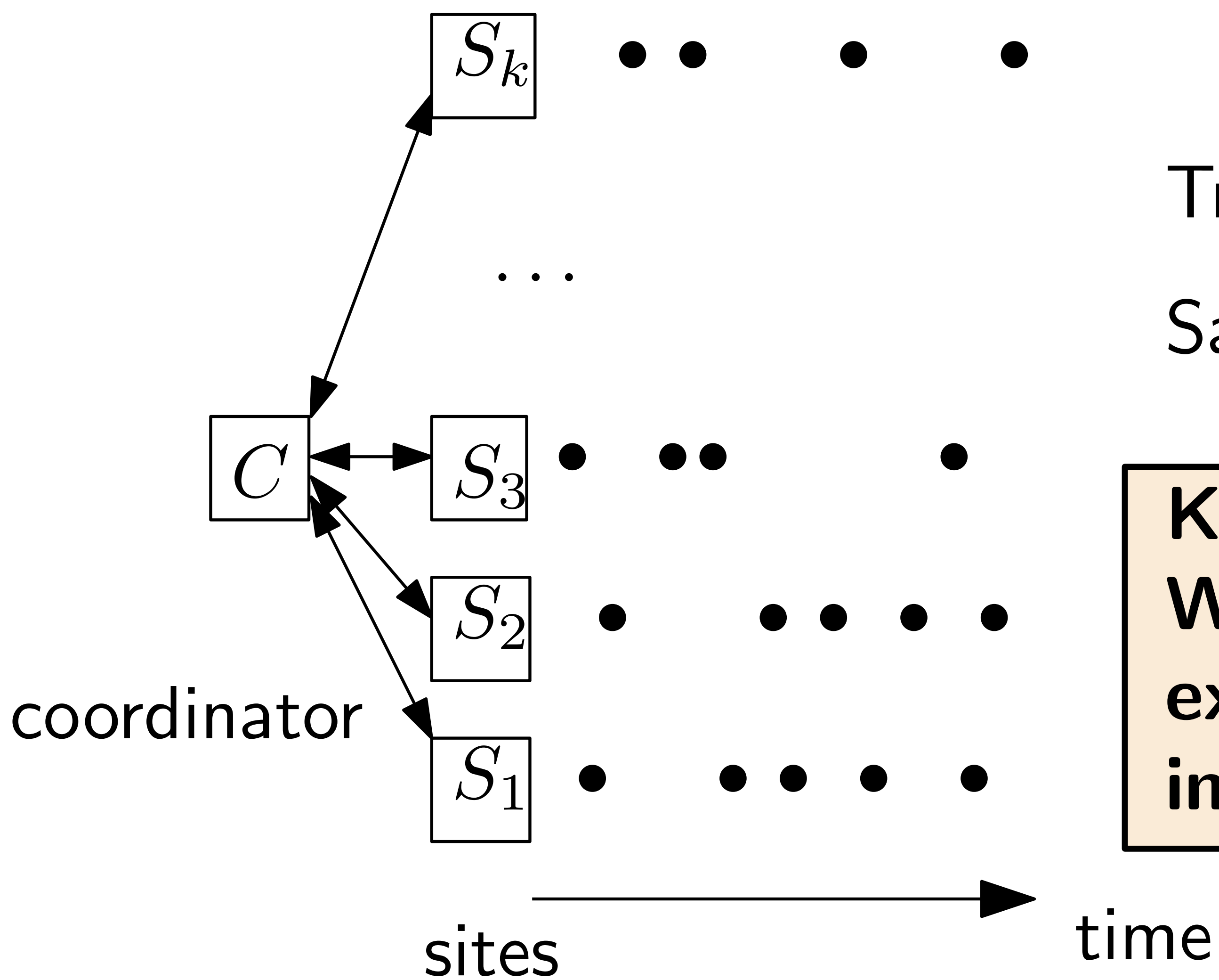

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Key observation:
We don't have to know the exact size of the population in order to sample!

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- Entropy [Arackaparambil, Brody, Chakrabarti, ICALP'08]
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- All of them are deterministic algorithms, or use randomized sketches as black boxes. And the trackings are "approximate".

Our results on random sampling

window	upper bounds	lower bounds
infinite	$O(k \log_{k/s} n + s \log n)$	$\Omega(k \log_{k/s} n + s \log n)$
sequence-based	$O(ks \log(w/s))$	$\Omega(ks \log(w/ks))$
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ISWoR

- The protocol
  - **Site:** always maintains an upper bound u (initialized to be 1) and lower bound l (initialized to be 0), and only sends those items with rank in the range $[l, u]$.

Rank: for each item coming, generate a random number in $[0, 1]$ as its rank.

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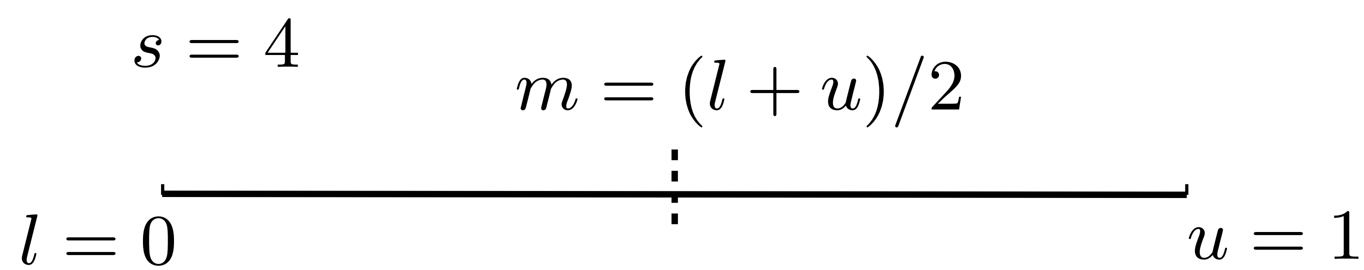

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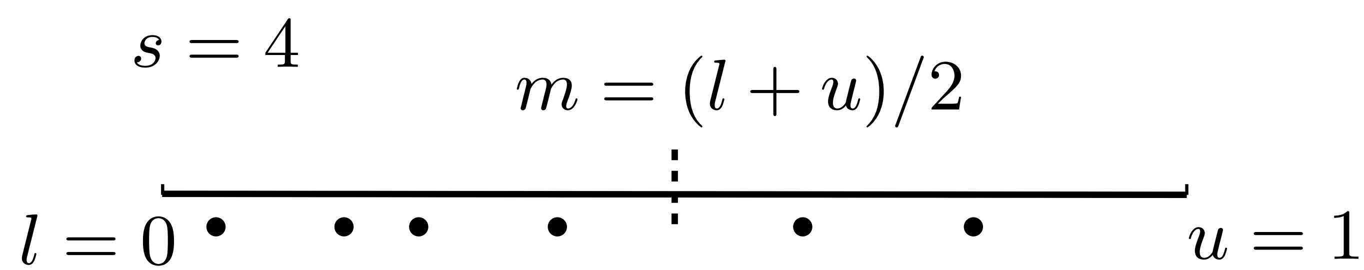

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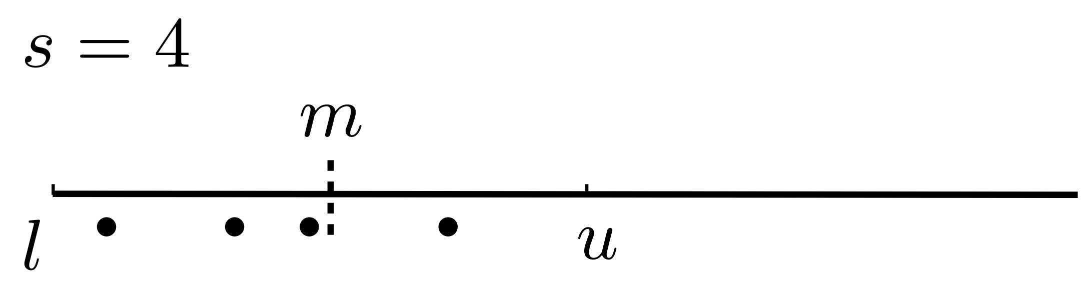

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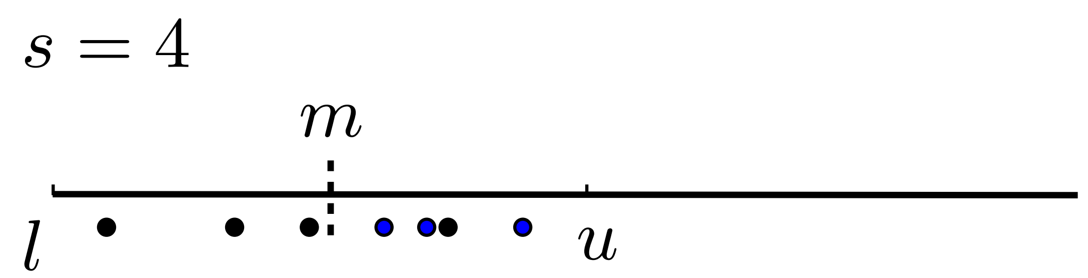

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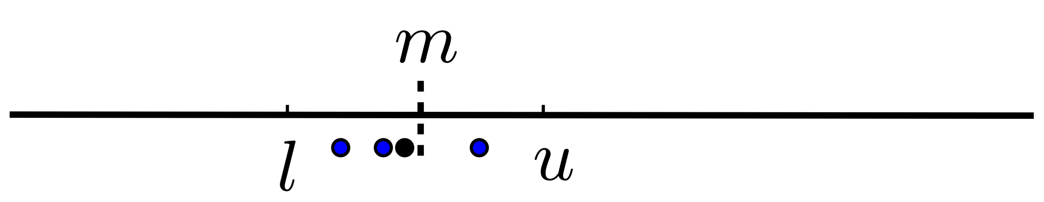

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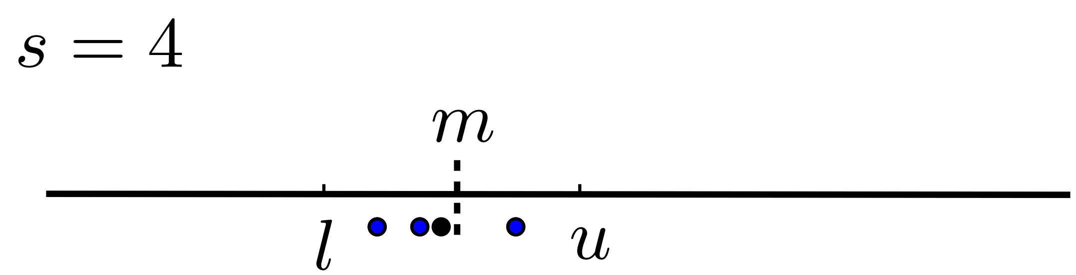

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Like Binary Search :)

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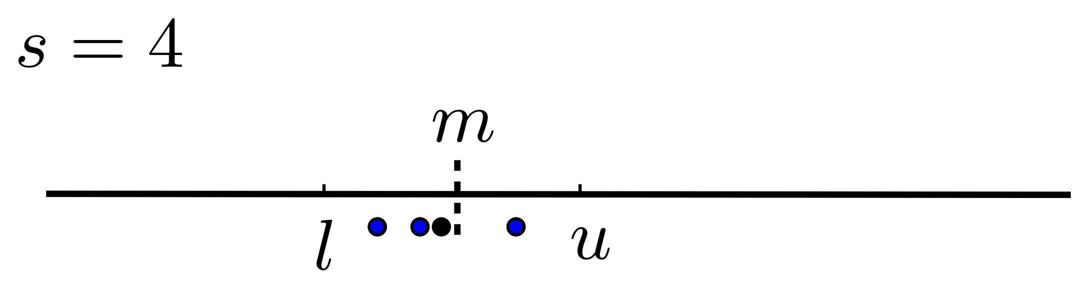

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Communication cost: $O((k + s) \log n)$

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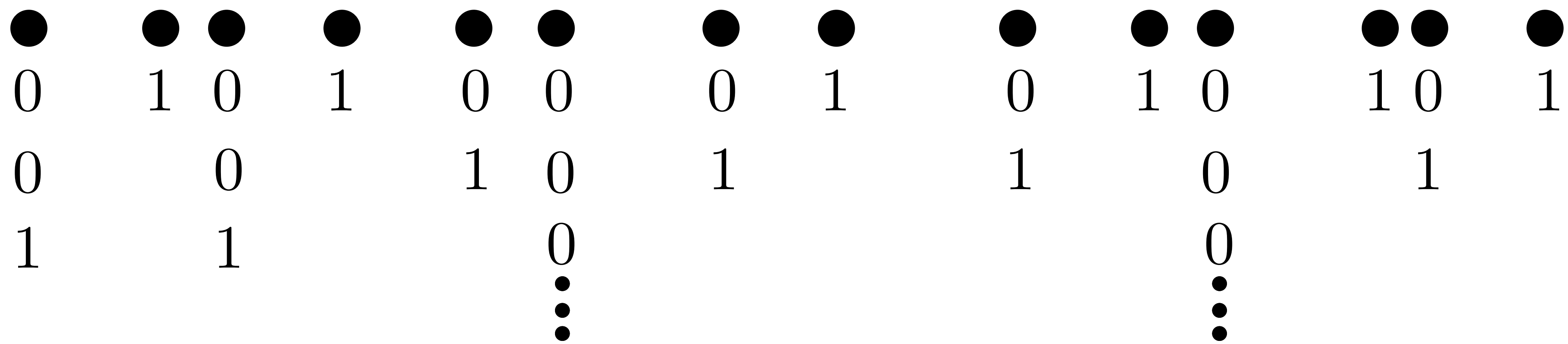

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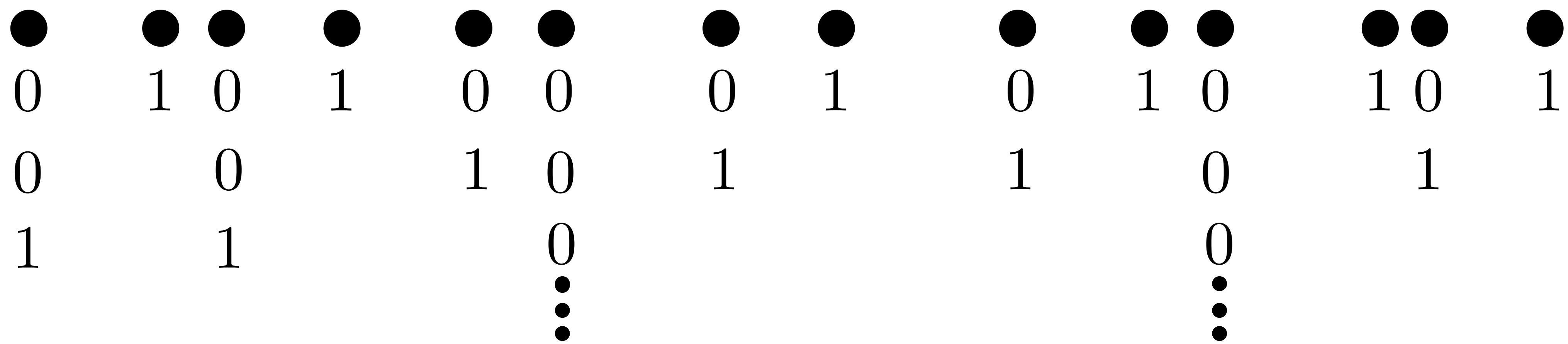

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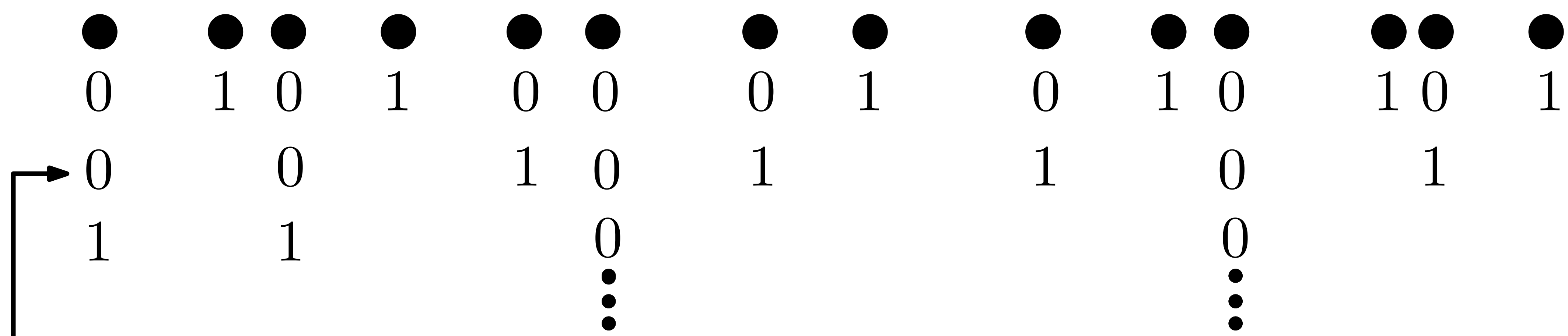

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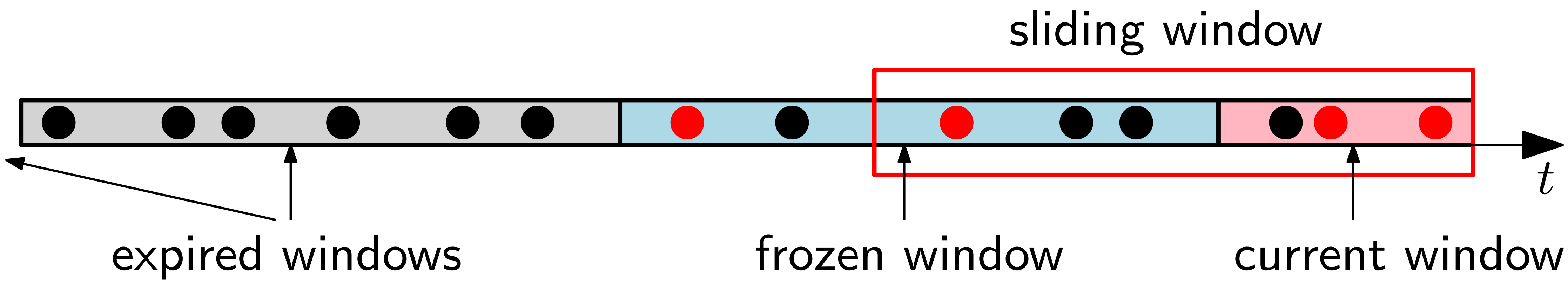
The coordinator could maintain a Bernoulli sample of size between s and $O(s)$

Sampling from a sliding window: Idea



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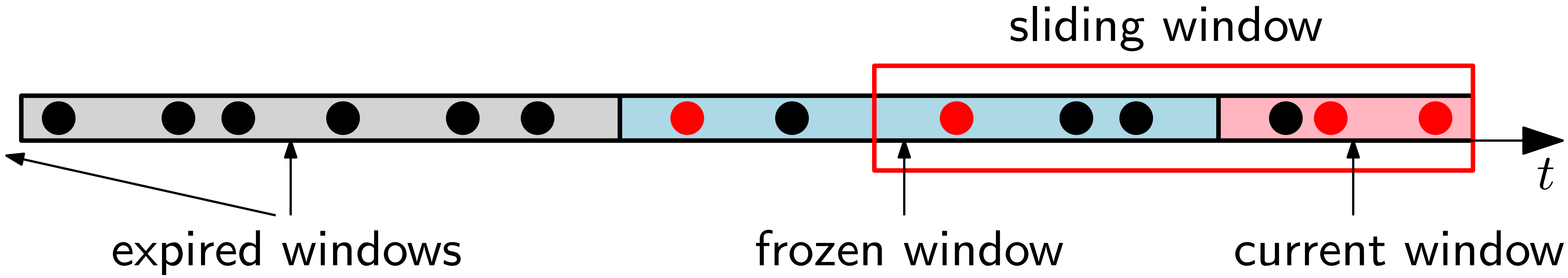

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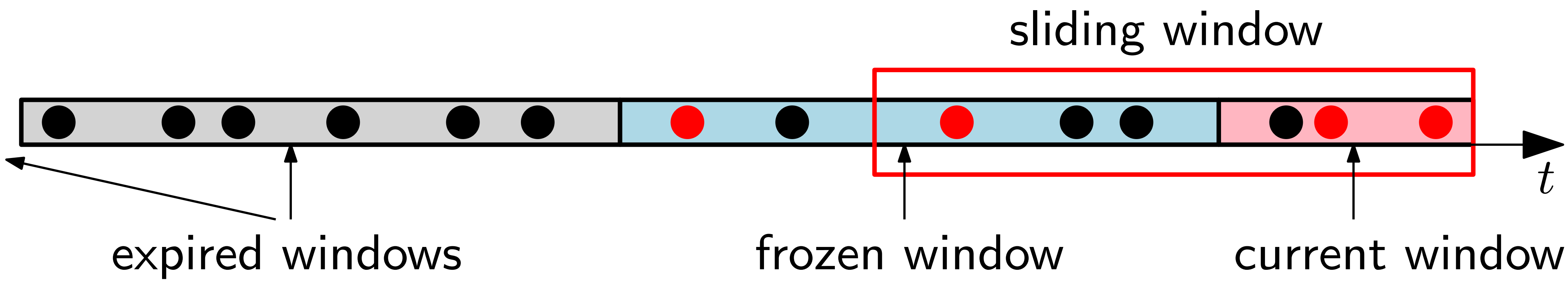

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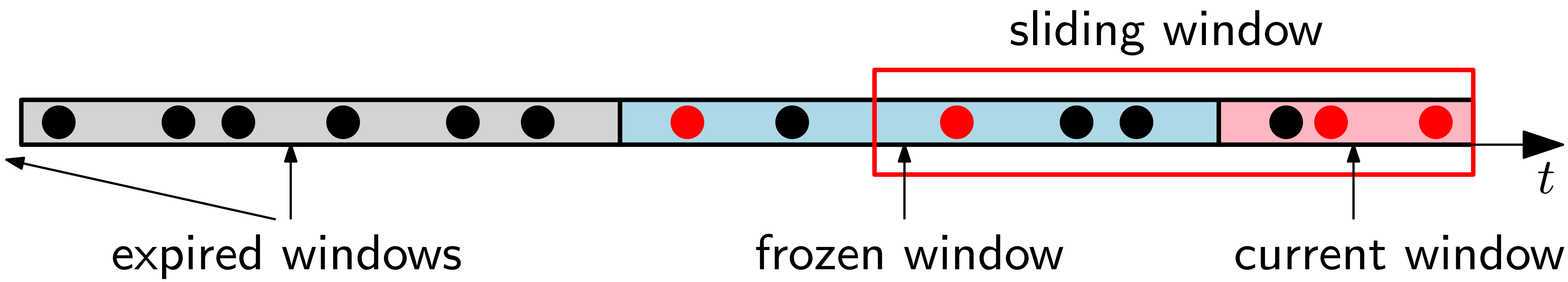

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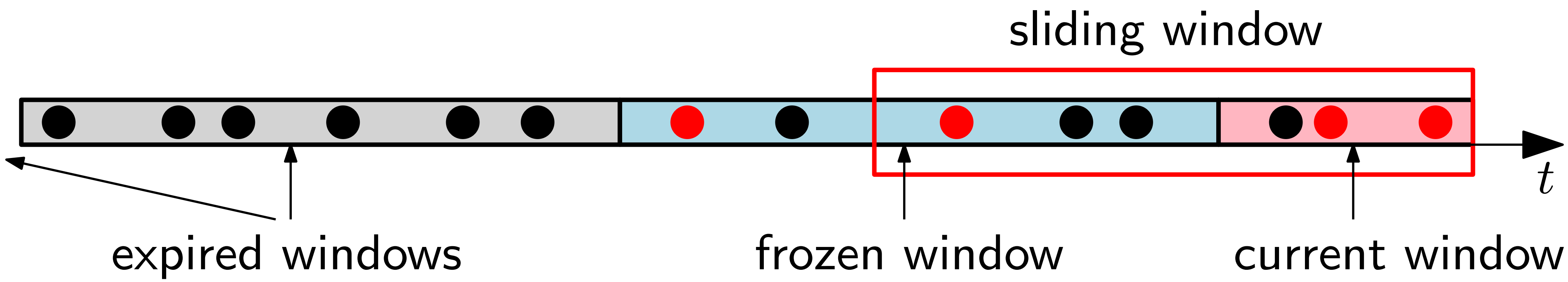

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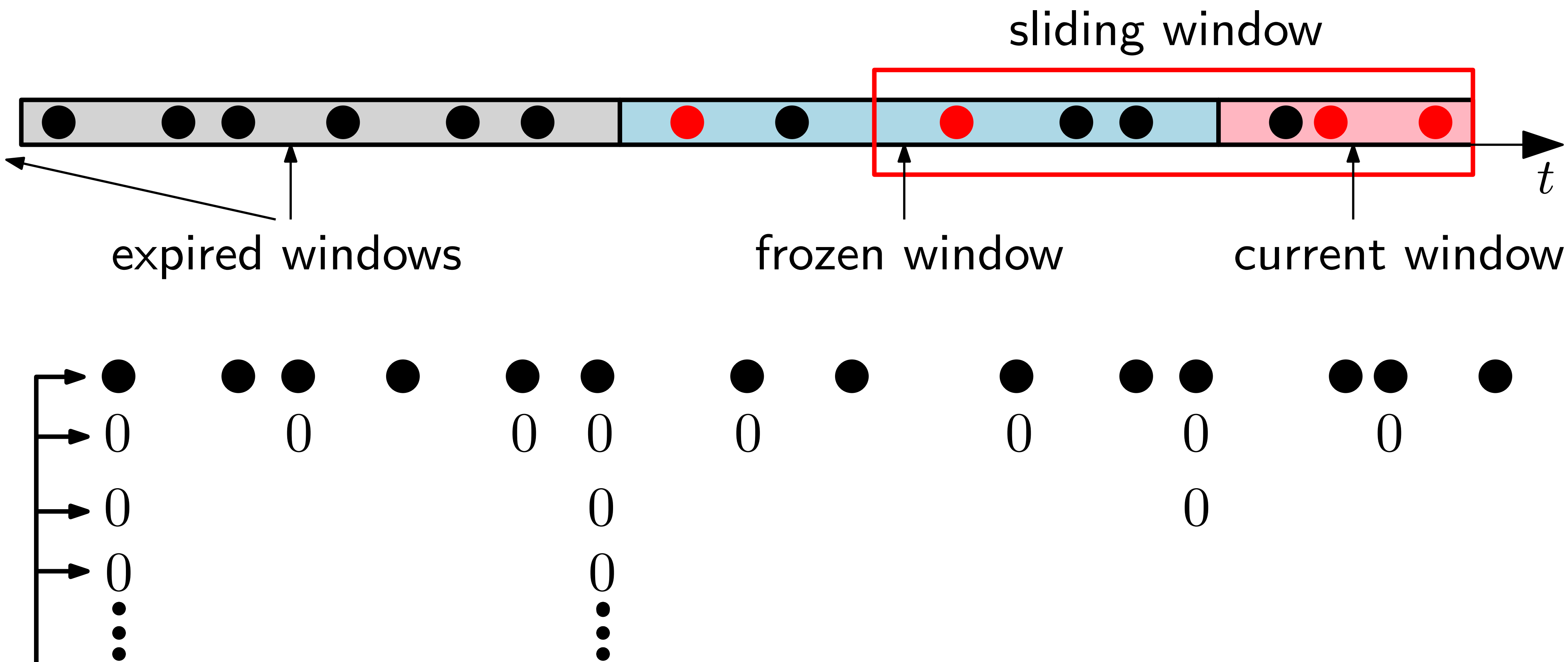
- The key issue: how to guarantee "both have sizes $\geq s$"?
  as items in the frozen window are expiring ...
  - Solution: In the frozen window, find a good sample rate such that the sample size $\geq s$.

Dealing with the frozen window



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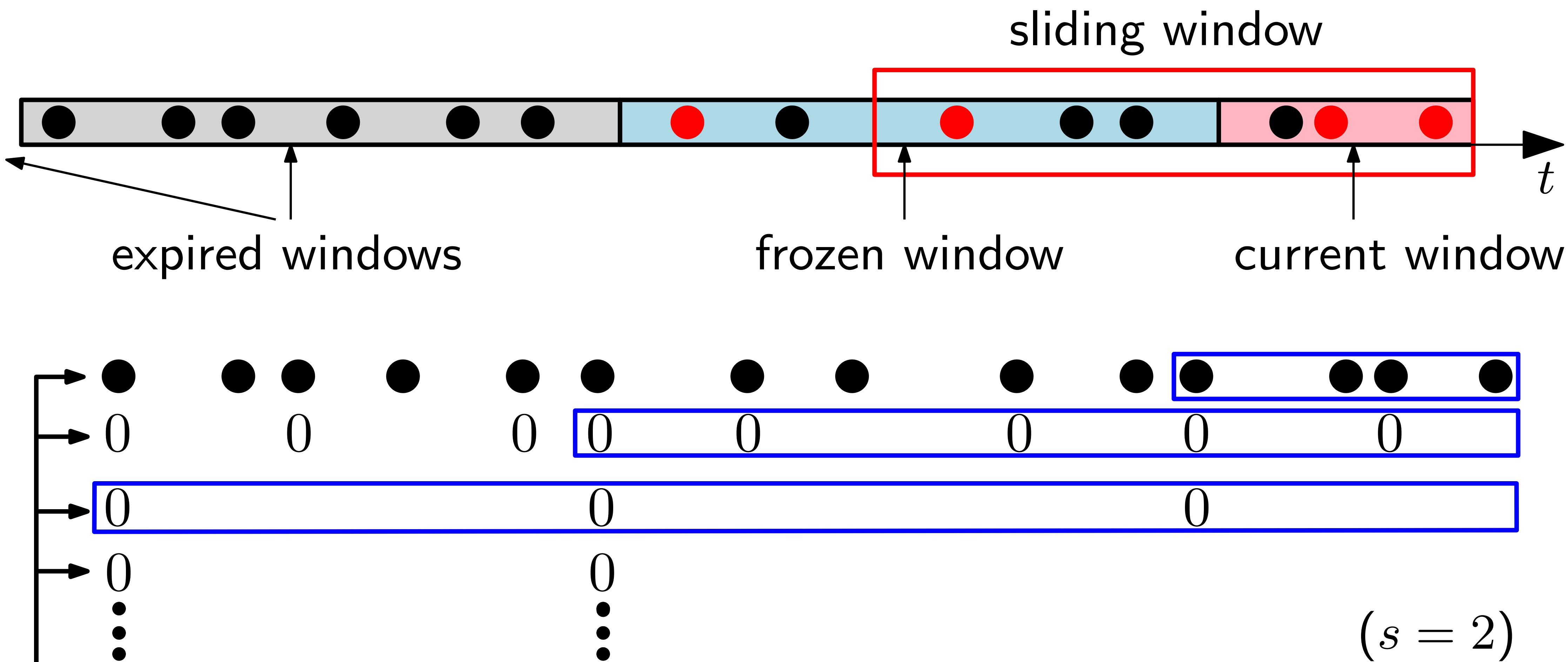

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Keep most recent sampled items in a level until s of them are also sampled at the next level. Total size: $O(s \log w)$

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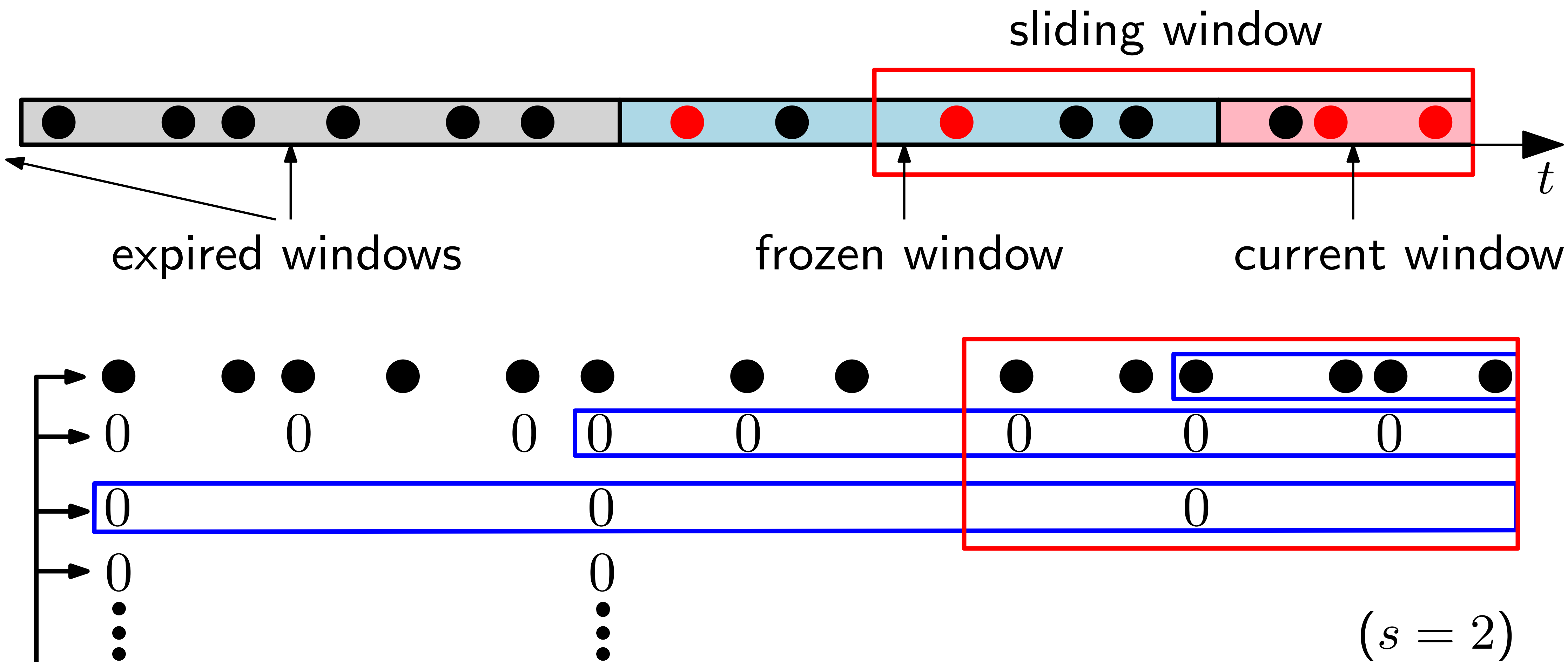

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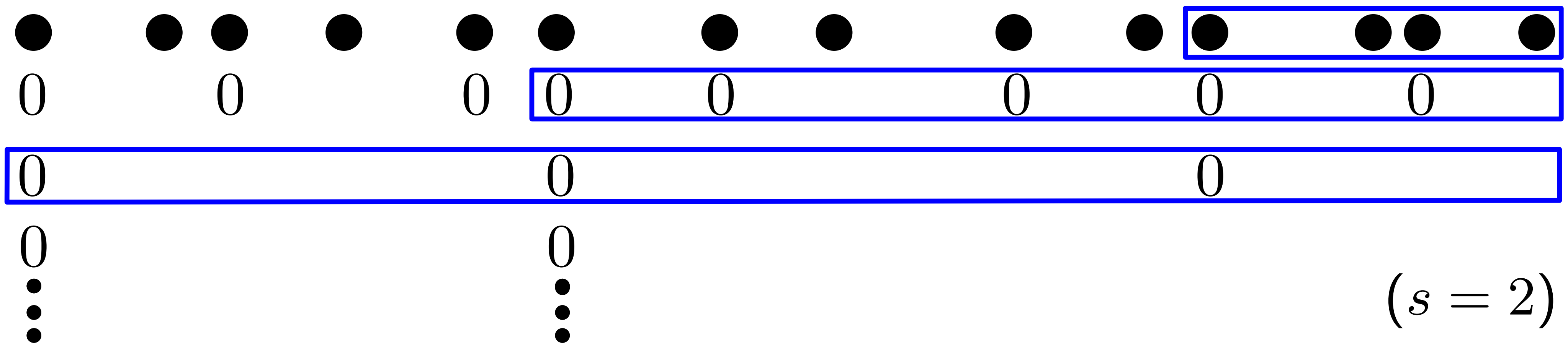
Keep most recent sampled items in a level until s of them are also sampled at the next level. Total size: $O(s \log w)$

Guaranteed: There is a blue window with $\geq s$ sampled items that covers the unexpired portion of the frozen window

Dealing with the frozen window: The algorithm



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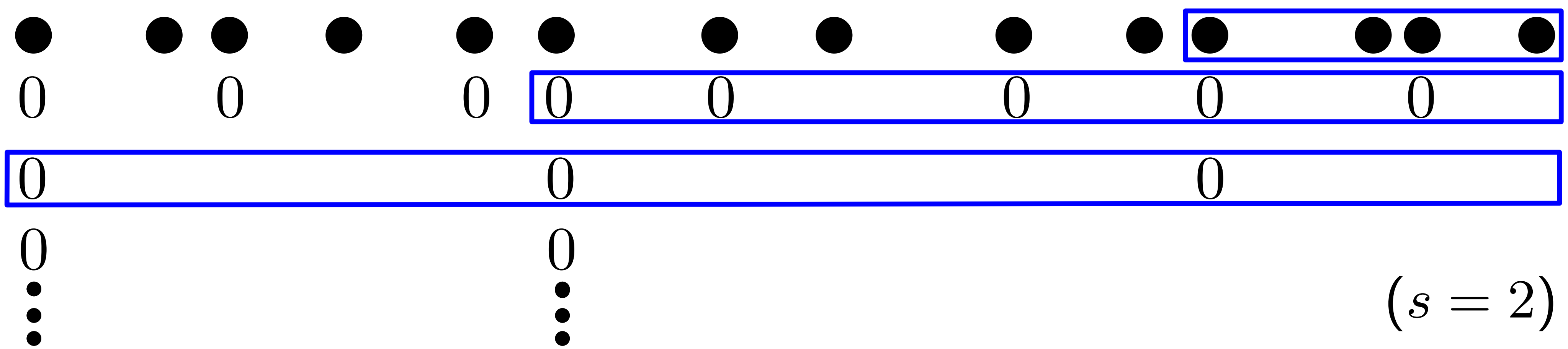

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- When the current window freezes
  - For each level, do a k-way merge to build the level of the global structure at the coordinator. Total communication $O((k+s)\log w)$

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  - Need some new ideas
- Processing time per item is another complicated issue for WR. Finally we can get $O(1)$ (but complicated).
- Experiments show that our algorithms work well.

Future directions

- Direct applications
  - Heavy hitters and quantiles can be tracked in $\tilde{O}(k + 1/\epsilon^2)$
    Beats deterministic bound $\tilde{\Theta}(k/\epsilon)$ for $k \gg 1/\epsilon$
  - Also for sliding windows
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- Other problems: range-counting, extent measures, etc.

Multiparty private message CC

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  - Public message
  - One-way communication

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- Now, it is the time!

The End

THANK YOU

Q and A