Lower Bound Techniques for Multiparty Communication Complexity

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Based on works with Jeff Phillips, Elad Verbin and David Woodruff
The multiparty number-in-hand (NIH) comm.
– A model for proving lower bounds

\[ x_1 = 010011 \quad x_2 = 111011 \]

\[ x_3 = 111111 \]

\[ x_k = 100011 \]

They want to jointly compute \( f(x_1, x_2, \ldots, x_k) \)

Goal: minimize total bits of communication
The multiparty number-in-hand (NIH) comm.

– A model for proving lower bounds

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Blackboard (BB): One speaks, everyone hears.

Message-Passing (MP): If \( x_1 \) talks to \( x_2 \), others cannot hear.

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Coordinator
Previously, for multiparty NIH comm. model

Well studied? YES and NO.
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Quite a few papers in BB model:

[Alon et al. ’96], [Bar-Yossef et al. ’04], . . .
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• Too easy?

• Lack of motivations?
Too easy?

- Could be more difficult than BB, since in MP each player observes only a piece of protocol transcript.
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- We do not want to analyze a $k$-dimensional matrix directly.

\[\begin{array}{c}
  \text{B} \\
  0 & 0 & 0 \\
  1 & 0 & 1 \\
\end{array}\]

A communication matrix of the 2-party AND function
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\begin{array}{ccc}
A & 0 & 1 \\
B & 0 & 1 & 0 \\
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\end{array}
\]

A communication matrix of the 2-party AND function

Even for the 2-party Gap-Hamming problem, where Alice has a $n$-bit vector and Bob has a $n$-bit vector, and they want to compute if the hamming distance of these two vectors is bigger than $n/2 + \sqrt{n}$ or less than $n/2 - \sqrt{n}$, it took researchers in this area about 10 years and dozens of papers (IW03, Woodruff04, . . ., Sherstov12) to fully understand its 2-dim communication matrix!
Lack of motivations?

- (Continuous) Distributed Monitoring
  communication $\rightarrow$ energy, bandwidth, ...
Lack of motivations? (cont.)

- Data Streams
  communication → space

Memory

CPU
Lack of motivations? (cont.)

- **Data Streams**
  
  communication → space

\[
P_1 \rightarrow P_2 \rightarrow \cdots \rightarrow P_k
\]
Lack of motivations? (cont.)

- **Distributed Computation for Big Data**
  
  communication \(\rightarrow\) bandwidth, ...

The **BSP model**.
E.g., Apache Giraph, Pregel.

The **MapReduce model**.
E.g., Hadoop, Google MapReduce.
Interesting problems

Statistical problems

- Frequency moments $F_p$
  - $F_0$: #distinct elements
  - $F_2$: size of self-join
- Heavy hitters
- Quantile
- Entropy
- ...
Interesting problems

Statistical problems

• Frequency moments $F_p$
  $F_0$: number of distinct elements
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Graph problems

• Connectivity
• Bipartiteness
• Counting triangles
• Matching
• Minimum spanning tree
• …
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Numerical linear algebra

- $L_p$ regression
- Low-rank approximation
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Numerical linear algebra

DB queries

- Conjunctive queries

Distributed learning

- Classifiers
  ...

Geometry problems

...
This talk

- We will talk lower bounds.
  Most lower bounds match the upper bounds.

- Introduce two general techniques that work in both BB model and MP model for proving lower bounds, with examples.

- Conclude with several future directions.
Our new ideas and techniques

- Ideas
  1. Reduce a $k$-player problem to a 2-player problem.
  2. Decompose a complicated problem to several simpler problems.
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- Concrete techniques
  1. **Symmetrization**
     With Phillips and Verbin, 2012
  2. **Composition**
     With Woodruff, 2012
Our new ideas and techniques

- **Ideas**
  1. Reduce a $k$-player problem to a 2-player problem.
  2. Decompose a complicated problem to several simpler problems.

- **In fact, we are**
  exploring the underlying patterns of communication matrices.

- **Concrete techniques**
  1. **Symmetrization**  
     Example: Graph Connectivity  
     With Phillips and Verbin, 2012
  2. **Composition**  
     Example: Distinct Elements  
     With Woodruff, 2012
Communication complexity

I choose not to introduce formal definitions/concepts of communication complexity. Just want to mention Yao’s lemma:

We will prove lower bounds for randomized protocols. To do that we will usually pick some input distribution, and then prove lower bounds for any deterministic protocol that succeeds w.pr 0.99 under that input distribution.
1st Technique: Symmetrization
Example: graph connectivity

\(k\) sites each holds a set of edges of a graph
Goal: compute whether the graph is connected.

A trivial UB: \(O(nk \log n)\) bits.
We prove that this is tight up to a log factor.
Set disjointness (2-DISJ)

\[ X \subseteq \{1, \ldots, n\} \]
\[ Y \subseteq \{1, \ldots, n\} \]

\[ X \cap Y = \emptyset? \]
Set disjointness (2-DISJ)

\[ X \subseteq \{1, \ldots, n\} \quad Y \subseteq \{1, \ldots, n\} \]

\[ X \cap Y = \emptyset? \]

Exists a hard distribution \( \mu_\beta \), under which

\[ |X \cap Y| = 1 \text{ (YES instance) w.p. } \beta \text{ and} \]
\[ |X \cap Y| = 0 \text{ (NO instance) w.p. } 1 - \beta. \]

**Lemma:** (Generalization of [Razborov '90, BJKS '04])

\[ E[CC_{\mu_\beta}^{\beta/100}(2\text{-DISJ})] = \Omega(n) \]
2-DISJ ⇒ $k$-CONN

Alice and Bob want to solve the 2-DISJ $(X, Y) \sim \mu_\beta$. (set $\beta = 1/k^2$)
⇒ running a protocol for $k$-CONN on:

Alice plays a random site $S_I$ with input $X_I = X$.
Bob plays other $k-1$ sites, and constructs inputs for them using $Y$: it choose $X_i \sim \mu_\beta | Y$ ($i \neq I$).
Note: $X_1, \ldots, X_k$ are i.i.d. conditioned on $Y$.

Vertices $u_1, \ldots, u_k, v_1, \ldots, v_n$. An edge $(u_i, v_j)$ exists iff $X_{i,j} = 1$.
Let $\nu$ be the distribution of the graph.
2-DISJ $\Rightarrow k$-CONN (cont.)

$v_j | j \in [r] \setminus Y$

$v_j | j \in Y$

$(u_i, v_j)$ exists if and only if $X_{i,j} = 1$

$X_I \cap Y = \emptyset$?
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Correctness: Solving $k$-CONN w.pr. $1 - \beta^{1.5}$ under $\nu$ also solves 2-DISJ w.pr. $1 - \beta/100$ under $\mu_\beta$. 
2-DISJ $\Rightarrow k$-CONN

$$\mathbb{E}[\text{CC}_{\mu_{\beta}}^{\beta/100}(2\text{-DISJ})] = O\left(\frac{1}{k}\right) \cdot \text{CC}_{\nu}^{\beta^{1.5}}(k\text{-CONN})$$

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A useful message

We can show trivial UBs are tight for many statistical and graph problems in the MP model (e.g., #distinct elements, bipartiteness, cycle/triangle-freeness, ...), when exact answers are wanted. (Woodruff, Z, 2013a)

Approximation is often necessary if we want comm. efficient protocols.
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Example:
Computing the diameter of a graph exactly needs $\Omega(kn^2)$ bits (when $m = n^2$) in the MP model.

However, there is a protocol that computes the diameter up to an additive error of 2 using $\tilde{O}(kn^{3/2})$ bits.
Other problems using symmetrization

- $k$-bitwise-MAJ
- $k$-bitwise-OR/AND/XOR

With applications in
  - Distinct elements reporting
  - $\varepsilon$-kernels (geometry)
  - Heavy-hitters

- Testing bipartiteness,
  - Testing cycle
  - Testing triangle-freeness,
  ...
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Key:

Find the right 2-player problem for reduction
2nd Technique: Composition
Example – the $F_0$ (distinct elements) problem

$k$ sites each holds a set $X_i$ ($i \in \{1, 2, \ldots, k\}$).
Goal: compute $\#\text{distinct elements}(\bigcup_{i=1}^{k} X_i)$ up to a $(1 + \varepsilon)$-approx.
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Current best UB: $\tilde{O}(k/\varepsilon^2)$
[Cormode, Muth, Yi. 2008]
Holds in MP model
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Previous LB: $\Omega(k)$
And $\Omega(1/\varepsilon^2)$
Direct reduction from Gap-Hamming
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Direct reduction from Gap-Hamming

The proof framework

Step 1: Find two modular problems $k$-GAP-MAJ and 2-DISJ of simpler structures s.t.

$F_0$ can be thought as a composition of them.

Step 2: Analyze the complexities of $k$-GAP-MAJ and 2-DISJ.

Step 3: Compose two modular problems so that:

\[ \text{Complexity}(F_0) = \text{Complexity}(k\text{-GAP-MAJ}) \times \text{Complexity}(2\text{-DISJ}). \]
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The proof presented here is for a special case when $k = 2/\varepsilon^2$. 
\(k\)-GAP-MAJ

\(k\) sites each holds a bit \(Z_i\) chosen uniform at random from \(\{0, 1\}\)

Goal: compute

\[
\text{\(k\)-GAP-MAJ}(Z_1, Z_2, \ldots, Z_k) = \begin{cases} 
0, & \text{if } \sum_{i \in [k]} Z_i \leq k/2 - \sqrt{k}, \\
1, & \text{if } \sum_{i \in [k]} Z_i \geq k/2 + \sqrt{k}, \\
\text{don't care}, & \text{otherwise},
\end{cases}
\]
$k$-GAP-MAJ

$k$ sites each holds a bit $Z_i$ chosen uniform at random from $\{0, 1\}$

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Lemma: Any protocol $\Pi$ that computes $k$-GAP-MAJ correctly w.p. 0.9999 has to learn $\Omega(k)$ $Z_i$’s well, that is,

$$H(Z_i \mid \Pi) \leq H_b(1/100)$$

for $\Omega(k)$ of $i$.

In other words: $I(\Pi; Z_1, \ldots, Z_k) = \Omega(k)$. 
Set disjointness ($2$-DISJ)

$X \subseteq \{1, \ldots, n\}$

$X \cap Y = \emptyset$?

$Y \subseteq \{1, \ldots, n\}$
Set disjointness (2-DISJ)

X \cap Y = \emptyset?

\begin{align*}
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\end{align*}

Exists a hard distribution \( \mu_\beta \), under which
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\end{align*}

**Lemma:** (Generalization of [Razborov ’90, BJKS ’04])
\[ \mathbb{E}\left[\text{CC}_{\mu_\beta}^{\beta/100} (2\text{-DISJ})\right] = \Omega(n) \]
The proof sketch

 coordinator \[ C \]

 sites \[ Y \]

 \[ (X_i, Y) \sim \mu_{1/2} \] (hard input dist. for DISJ) for each \( i \in [k] \)
The proof sketch

coordinator $C$

2-DISJ

sites $S_1, S_2, S_3, \ldots, S_k$ $X_1, X_2, X_3, \ldots, X_k$

$Y$

$Z_i = |X_i \cap Y| \begin{cases} 1 & \text{w.p. } 1/2 \\ 0 & \text{w.p. } 1/2 \end{cases}$

$(X_i, Y) \sim \mu_{1/2}$ (hard input dist. for DISJ) for each $i \in [k]$
The proof sketch

 coordinator
 2-DISJ

 sites

  \[ S_1 \quad S_2 \quad S_3 \quad \cdots \quad S_k \]
  \[
  \begin{aligned}
  X_1 & \quad X_2 & \quad X_3 & \quad \cdots & \quad X_k \\
  \end{aligned}
  \]

  \( (X_i, Y) \sim \mu_{1/2} \) (hard input dist. for DISJ) for each \( i \in [k] \)

  \[ F_0(X_1, X_2, \ldots, X_k) \iff k\text{-GAP-MAJ}(Z_1, Z_2, \ldots, Z_k) \]

  \( (Z_i = |X_i \cap Y|) \)

  \( \iff \) learn \( \Omega(k) \) \( Z_i \)'s well

  \( \iff \) need \( \Omega(nk) \) bits

  (using an embedding argument)
The proof sketch

coordinator $[C]$

2-DISJ

sites $[S_1]$ $[S_2]$ $[S_3]$ $\cdots$ $[S_k]$ $Y$

$X_1$ $X_2$ $X_3$ $X_k$

$Z_i = |X_i \cap Y| \begin{cases} 1 \text{ w.p. } 1/2 \\ 0 \text{ w.p. } 1/2 \end{cases}$

$(X_i, Y) \sim \mu_{1/2}$ (hard input dist. for DISJ) for each $i \in [k]$

$F_0(X_1, X_2, \ldots, X_k) \iff k$-GAP-MAJ($Z_1, Z_2, \ldots, Z_k$)

$(Z_i = |X_i \cap Y|)$

$\implies$ learn $\Omega(k)$ $Z_i$’s well

$\implies$ need $\Omega(nk)$ bits (using an embedding argument)

For the reduction set universe $n = \Theta(1/\varepsilon^2)$

we get $\Omega(k/\varepsilon^2)$ LB.

Q.E.D.
Other problems using composition

- Frequency moments $F_p \ (p > 1)$
- Entropy
- $\ell_p$
- Heavy-hitters
- Quantile
Other problems using composition

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Key:

Find the right modular problems
Furture directions
Future directions – Problem space

The current understandings

<table>
<thead>
<tr>
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<th>linear algebra</th>
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Concretely, e.g.,
What is the CC of computing a const approx of the size of the matching?
Communication complexity of distributed property testing can also be used for proving LBs for traditional property testing, by extending a framework by Blais et al. 2011.
Future directions – Problem space (cont.)

- Communication complexity of distributed property testing.
  Communication complexity of distributed property testing can also be used for proving LBs for traditional property testing, by extending a framework by Blais et al. 2011.

- Communication complexity of relations
  (instead of functions, useful for database queries)
Future directions – Problem space (cont.)

- Communication complexity of distributed property testing.
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- Communication complexity of relations (instead of functions, useful for database queries)

- Need to understand more primitive problems (building blocks), e.g., breadth-first search (BFS), pointer-chasing, etc.
  So far building blocks are quite limited: mainly 2-DISJ, 2-GAP-HAMMING, $k$-GAP-MAJ, $k$-OR and their variants.
Future directions – Model/Technique space

- Can we relax or generalize the symmetrization technique?
  E.g., do not require “totally symmetric distributions”
Future directions – Model/Technique space

- Can we relax or generalize the symmetrization technique? E.g., do not require “totally symmetric distributions”
- Consider round complexity (practical needs).
- Consider players with limited space (practical needs).
The end

THANK YOU

Questions?