B669 Sublinear Algorithms for Big Data

Qin Zhang
Now about the Big Data

- Big data is everywhere
  - Walmart: over 2.5 petabytes of sales transactions
  - Google: an index of over 19 billion web pages
  - Facebook: over 40 billion of pictures
  - …
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- Magazine covers
  - Nature ’06
  - Nature ’08
  - CACM ’08
  - Economist ’10
Source and Challenge

- **Source**
  - Retailer databases
  - Logistics, financial & health data
  - Social network
  - Pictures by mobile devices
  - Internet of Things
  - New forms of scientific data
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■ Challenge
  • Volume
  • Velocity
  • Variety (Documents, Stock records, Personal profiles, Photographs, Audio & Video, 3D models, Location data, ...)
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  - Variety (Documents, Stock records, Personal profiles, Photographs, Audio & Video, 3D models, Location data, ...)
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- The data is too big to be stored in a single machine. What can we do if we do not want to throw them away?
  - Store in multiple machines, which collaborate via communication Sublinear in communication
What do we mean by “sublinear”? 

Time/space/communication spent is $o($input size$)$
Concretely, theory folks talk about the following ... 

- Sublinear time algorithms
  - Sublinear time approximation algorithms
  - Property testing (not in this course)
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  - Data stream algorithms
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- **Sublinear communication algorithms**
  - Multiparty communication protocols/algorithms
    (particular models: MapReduce, BSP, …)
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- Sublinear space algorithms
  - Data stream algorithms

- Sublinear communication algorithms
  - Multiparty communication protocols/algorithms
    (particular models: MapReduce, BSP, ...)

- Sublinear I/O algorithms (not in this course)
  - External memory data structures/algorithms
Given a social network graph, we want to compute its average degree. (i.e., the average # of friends people have in the network)

Can we do it without querying the degrees of all nodes? (i.e., asking everyone)
Why hard? You can’t see everything in sublinear time!

- Computing exact average degree is impossible without querying at least $n - 1$ nodes ($n$: \# total nodes).

So our goal is to get a $(1 + \epsilon)$-approximation w.h.p. ($\epsilon$ is a very small constant, e.g., 0.01)
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- Can we simply use sampling?
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- Can we simply use sampling?

No, it doesn’t work. Consider the star, with degree sequence \((n - 1, 1, \ldots, 1)\).
Computing exact average degree is impossible without querying at least \( n - 1 \) nodes (\( n: \# \text{total nodes} \)).

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So can we do anything non-trivial? (think about it, and we will discuss later in the course)
Sublinear in space

- **The data stream model** (Alon, Matias and Szegedy 1996)
Sublinear in space

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- **Applications**
  - Internet Router.
  - Stock data, ad auction, flight logs on tapes, etc.

The router wants to maintain some statistics on data. E.g., want to detect anomalies for security.
Why hard? You do see everything but then “forget”!

- **Game 1**: A sequence of numbers
Why hard? You do see everything but then “forget”!

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52
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45
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  18
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\[ 23 \]
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17
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- **Game 1**: A sequence of numbers

41
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![33]
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- **Game 1**: A sequence of numbers

  29
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- **Game 1**: A sequence of numbers

49
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- **Game 1**: A sequence of numbers

  12
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![35]
Why hard? You do see everything but then “forget”!

- **Game 1**: A sequence of numbers

  **Q**: What’s the *median*?
Why hard? You do see everything but then “forget”!

- **Game 1**: A sequence of numbers

  Q: What’s the median?

  A: 33

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- **Game 2**: Relationships between Alice, Bob, Carol, Dave, Eva and Paul

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  Alice and Bob become friends

- **Game 1**: A sequence of numbers

  Q: What’s the median?

  A: 33

- **Game 2**: Relationships between Alice, Bob, Carol, Dave, Eva and Paul

  Carol and Eva become friends

- **Game 1:** A sequence of numbers

  Q: What’s the **median**?

  A: 33

- **Game 2:** Relationships between Alice, Bob, Carol, Dave, Eva and Paul

  Eva and Bob become friends

- **Game 1**: A sequence of numbers

  Q: What’s the median?

  A: \( \text{33} \)

- **Game 2**: Relationships between Alice, Bob, Carol, Dave, Eva and Paul

  Dave and Paul become friends

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  Alice and Paul become friends

- **Game 1**: A sequence of numbers

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- **Game 2**: Relationships between Alice, Bob, Carol, Dave, Eva and Paul

  Eva and Bob unfriends

- **Game 1**: A sequence of numbers

  Q: What’s the median?

  A: 33

- **Game 2**: Relationships between Alice, Bob, Carol, Dave, Eva and Paul

  Alice and Dave become friends

- **Game 1**: A sequence of numbers

  Q: What’s the median?

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- **Game 2**: Relationships between Alice, Bob, Carol, Dave, Eva and Paul

  Bob and Paul become friends

- **Game 1**: A sequence of numbers

  Q: What’s the median?

  A: 33

- **Game 2**: Relationships between Alice, Bob, Carol, Dave, Eva and Paul

  Dave and Paul unfriends

- **Game 1**: A sequence of numbers

  Q: What’s the median?

  A: 33

- **Game 2**: Relationships between Alice, Bob, Carol, Dave, Eva and Paul

  Dave and Carol become friends

- **Game 1**: A sequence of numbers
  
  **Q**: What’s the **median**?
  
  **A**: 33

- **Game 2**: Relationships between Alice, Bob, Carol, Dave, Eva and Paul
  
  **Q**: Are Eva and Bob connected by friends?

- **Game 1:** A sequence of numbers
  
  **Q:** What’s the **median**?
  
  **A:** 33

- **Game 2:** Relationships between Alice, Bob, Carol, Dave, Eva and Paul
  
  **Q:** Are Eva and Bob connected by friends?
  
  **A:** YES. Eva ⇔ Carol ⇔ Dave ⇔ Alice ⇔ Bob

- **Game 1**: A sequence of numbers
  
  **Q**: What’s the median?
  
  **A**: 33

- **Game 2**: Relationships between Alice, Bob, Carol, Dave, Eva and Paul
  
  **Q**: Are Eva and Bob connected by friends?
  
  **A**: YES. Eva ⇔ Carol ⇔ Dave ⇔ Alice ⇔ Bob

- Have to allow approx/randomization given a small memory.
Sublinear in communication

The model

$x_1 = 010011$

$x_2 = 111011$

$x_3 = 111111$

$x_k = 100011$

They want to jointly compute $f(x_1, x_2, \ldots, x_k)$ (e.g., $f$ is $\#$ distinct ele)

Goal: minimize total bits of communication
Sublinear in communication

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  \[ x_1 = 010011 \]
  \[ x_2 = 111011 \]
  \[ x_3 = 111111 \]
  \[ x_k = 100011 \]

  They want to jointly compute \( f(x_1, x_2, \ldots, x_k) \) (e.g., \( f \) is the number of distinct elements)

  Goal: minimize total bits of communication

- **Applications**

  etc.
Why hard? You do not have a global view of the data.

Let’s think about the **graph connectivity** problem: 
$k$ machine each holds a set of edges of a graph.

Goal: compute whether the graph is connected.
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A trivial solution: each machine sends a **local spanning tree** to the first machine. Cost $O(kn \log n)$ bits.
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Can we do better, e.g., $o(kn)$ bits of communication?
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Can we do better, e.g., $o(kn)$ bits of communication?

What if the graph is **node partitioned** among the $k$ machines? That is, each node is stored in 1 machine with all adjacent edges.
Problems

Statistical problems

• Frequency moments $F_p$
  $F_0$: #distinct elements
  $F_2$: size of self-join
• Heavy hitters
• Quantile
• Entropy
• ...
Problems

Statistical problems

- Frequency moments $F_p$
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Graph problems

- Connectivity
- Bipartiteness
- Counting triangles
- Matching
- Minimum spanning tree
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Numerical linear algebra

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- Bipartiteness
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- $L_p$ regression
- Low-rank approximation
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Problems

Statistical problems
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- Connectivity
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- ...

DB queries
- Conjunctive queries

Strings
- Edit distance
- Longest increasing sequence

Geometry problems
- Clustering
- Earth-Mover Distance
- ...

...
Example: random sampling in data stream
A toy example: Reservoir Sampling

**Tasks:** Find a **uniform sample** from a stream of unknown length, can we do it in $O(1)$ space?
A toy example: Reservoir Sampling

**Tasks:** Find a **uniform sample** from a stream of unknown length, can we do it in $O(1)$ space?

**Algorithm:** Store 1-st item. When the $i$-th ($i > 1$) item arrives

- With probability $1/i$, replace the current sample;
- With probability $1 - 1/i$, throw it away.
A toy example: Reservoir Sampling

Tasks: Find a uniform sample from a stream of unknown length, can we do it in $O(1)$ space?

Algorithm: Store 1-st item. When the $i$-th ($i > 1$) item arrives

- With probability $1/i$, replace the current sample;
- With probability $1 - 1/i$, throw it away.

Correctness: each item is included in the final sample w.p.

$$\frac{1}{i} \times (1 - \frac{1}{i+1}) \times \ldots \times (1 - \frac{1}{n}) = \frac{1}{n} \quad (n: \text{total \# items})$$

Space: $O(1)$
Tasks: Find a uniform sample from the last $w$ items.
Maintain a sample for Sliding Windows

**Tasks:** Find a uniform sample from the last \( w \) items.

**Algorithm:**
- For each \( x_i \), we pick a random value \( v_i \in (0, 1) \).
- In a window \( < x_{j-w+1}, \ldots, x_j > \), return value \( x_i \) with smallest \( v_i \).
- To do this, maintain the set of all \( x_i \) in sliding window whose \( v_i \) value is minimal among subsequent values.
Tasks: Find a uniform sample from the last $w$ items.

Algorithm:
- For each $x_i$, we pick a random value $v_i \in (0, 1)$.
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- To do this, maintain the set of all $x_i$ in sliding window whose $v_i$ value is minimal among subsequent values.

Correctness: Obvious.

Space (expected): $1/w + 1/(w - 1) + \ldots + 1/1 = \log w$. 
Tentative course plan

Part 0 : Introductions
  – New models for Big Data, interesting problems
  – Basic probabilistic tools

Part 1 : Sublinear in space
  – Distinct elements, heavy hitters, $\ell_0$-sampling

Part 2 : Sublinear in communication
  – Connectivity, min-cut and sparsification

Part 3 : Sublinear in time
  – Average degree, minimum spanning tree

Part 4 : Random topics
  – E.g., distributed monitoring

Part 5 : Student presentations
Resources

- There is no textbook for the class. Reference for part of the course: lecture notes by Amit Chakrabarti

- Background on Randomized Algorithms:
  - **Probability and Computing**
    by Mitzenmacher and Upfal
    (Advanced undergraduate textbook)
  - **Randomized Algorithms**
    by Motwani and Raghavan
    (Graduate textbook)
Surveys

- Sketch Techniques for Approximate Query Processing by Cormode
- Data Streams: Algorithms and Applications by Muthukrishnan

Check course website for more resources
http://homes.soic.indiana.edu/qzhangcs/B669-17-fall-sublinear/index.html
Instructors

- **Instructor:** Qin Zhang  
  Email: qzhangcs@indiana.edu  
  Office hours: Wed. 4-5pm at LH430A

- **Associate Instructor:** Ruiyu Zhu  
  Email: rynzhu@gmail.com  
  Office hours: Mon. 4-5pm at Lindley Abyss
Grading

Assignments 40% : – Two homework assignments.
– Solutions should be typeset in LaTeX and submitted via Canvas.
– Will be a HW0 for practicing LaTeX (0 pt; −5 if not submitted)

Project 60% : Consists of the following components:
(details see course website)
1. Write a proposal.
2. Write a report.
3. Make a presentation.
4. Grade others’ presentations.

Final grade will be curved
A research-oriented course. Will be quite mathematical.

One is expected to know:
basics on algorithm design and analysis + basic probability.
e.g., have taken B403 “Introduction to Algorithm Design and Analysis” or equivalent courses.

I will NOT start with things like big-O notations, the definitions of expectation and variance, and hashing.
Is this a course good for my job hunting in industry?

Yes and No.

Yes, if you get to know some advanced (and easily implementable) algorithms for handling big data, that will certainly help. (e.g., Google interview questions)

But, this is a research-oriented course, and is NOT designed for teaching commercially available techniques.
Frequently asked questions

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  Yes and No.

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  But, this is a research-oriented course, and is NOT designed for teaching commercially available techniques.

- **I haven’t taken B403 “Introduction to Algorithm Design and Analysis” or equivalent courses. Can I take the course? Or, will this course fit me?**

  Generally speaking, this is an advanced algorithm course. It might be difficult if you do not have enough background (math + programming). So think carefully before taking this course!
Summary for today

- We have introduced three types of sublinear algorithms: in time, space, and communication.

- We have talked about why algorithmic design in these models/paradigms are difficult.

- We have discussed some simple problems.

- We have talked about the course plan and assessment.
Thank you!