A combinator library for MCMC kernels and proposals

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Motivation

MCMC samplers are complicated

$$\mathcal{A}\left(x^{(i)}, x^{\star}\right) = \min\left\{1, \frac{p(x^{\star})q\left(x^{(i)} \mid x^{\star}\right)}{p\left(x^{(i)}\right)q\left(x^{\star}|x^{(i)}\right)}\right\}$$

An Introduction to MCMC for Machine Learning, C. Andrieu et al., 2003

$$\mathcal{A}_{n\to m} = \min\left\{1, \frac{p(m, x_m^\star)}{p(n, x_n)} \times \frac{q(n \mid m)}{q(m \mid n)} \times \frac{q_{m\to n}(u_{m,n} \mid m, x_m^\star)}{q_{n\to m}(u_{n,m} \mid n, x_n)} \times \mathcal{J}_{f_{n\to m}}\right\}$$

An Introduction to MCMC for Machine Learning, C. Andrieu et al., 2003

$$\mathcal{A}_{split} = \min\left\{1, \frac{p(k+1, \mu_{k+1})}{p(k, \mu_k)} \times \frac{\frac{1}{k+1}}{\frac{1}{k}} \times \frac{1}{p(u_{n,m})} \times \mathcal{J}_{split}\right\}$$

An Introduction to MCMC for Machine Learning, C. Andrieu et al., 2003

$$\mathcal{A}_{merge} = \min\left\{1, \frac{p(k-1, \mu_{k-1})}{p(k, \mu_k)} \times \frac{\frac{1}{k-1}}{\frac{1}{k}} \times \mathcal{J}_{merge}\right\}$$

An Introduction to MCMC for Machine Learning, C. Andrieu et al., 2003

We would like predefined samplers

MCMC depends on “good” proposals

A 1-D target distribution

$$p(x) \propto 0.3 \exp(-0.2x^2) + 0.7 \exp(-0.2(x-10)^2)$$

A gaussian might work as a proposal

What is the optimal mean? Standard deviation?

We would like to easily experiment with different proposals on a sampler

A second look

$$p(x) \propto 0.3 \exp(-0.2x^2) + 0.7 \exp(-0.2(x-10)^2)$$

A mixture of gaussians might work even better

We would like to easily mix various proposals

Quick info

Proposal and target combinators

Kernel combinators

Making the random walk

Future work

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Future work

The library is written in Haskell

It extends Indiana's probabilistic programming system

Consists of three modules:

Distributions, Actions, Kernels

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Targets vs proposals

Targets have a density

```haskell
type Density a = a → Probability
```

```haskell
data Target a = T (Density a)
```

Proposals have a density and can be sampled from

```haskell
type Sample a = Rand → IO a
```

```haskell
data Proposal a =
       P (Density a) (Sample a)
```

Proposals

What proposals are already defined?

uniform, normal,

categorical

```
uniform [0,2,4] [3,5,7]

normal [0,1,4,7] (diag [2,2,2,2])

categorical
["pizza", "sushi", "curry"]
[0.7, 0.9, 0.3]
```

How do we create our own proposals?

Mix existing proposals

```
proposalMix ::
 [Proposal a] → [Double] → Proposal a
```

```
gMix = let g1 = normal [0,0] (diag [1,1])
           g2 = normal [5,5] (diag [2,2])
       in proposalMix [g1, g2] [0.3, 0.7]
```

Increase dimensions of existing proposals

Use `updateNth`, `updateBlock`

```
updateNth :: Int
          → ([a] → Proposal [a])
```

```
condProposal =
updateNth
4
(λy → normal y [[1]])
```

Update component a_4

for a point $(a_1, a_2, a_3, a_4, \ldots, a_n)$

```
updateBlock :: Int
            → Int
            → ([a] → Proposal [a])
            → ([a] → Proposal [a])
```

```
condProposal =
updateBlock
1
2
(λy → normal y (diag [1,1])
```

Update components a_1, a_2
for a point $(a_1, a_2, \ldots, a_n)$

Construct our own

using `makeProposal`

```
customProposal =
let density = ...
    sample = ...
in makeProposal density sample
```

Targets

How do we create targets?

Use `makeTarget`

```
t1 = makeTarget $
     λ[x] → 0.3 * exp (-0.2*x*x)
```

Mix existing targets, using `targetMix`

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What is a kernel?

A function that, when given a target and conditional proposal, performs MCMC steps

```
type Step x = Rand → x → IO x

type Kernel x a =
  Target a →
  (a → Proposal a) →
  Step x
```

What kernels are already defined?

metropolisHastings,
simulatedAnnealing

How do we define our own steps?

Mix some steps

```
mixSteps :: [Step x]
         → [Double]
         → Step x
```

```haskell
mhMix = let mh = metropolisHastings
            mh1 = mh target proposal1
            mh2 = mh target proposal2
        in mixSteps [mh1, mh2] [0.7, 0.3]
```

Cycle through a kernel

```
cycleKernel :: Kernel x a
            → Target a
            → [a → Proposal a]
            → Step x
```

```
mhCycle =
cycleKernel
metropolisHastings
target
[p1, p2, p3]
```

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```
walk :: Step x
     → x
     → Int
     → Rand
     → Action x a
     → IO a
```

Start state

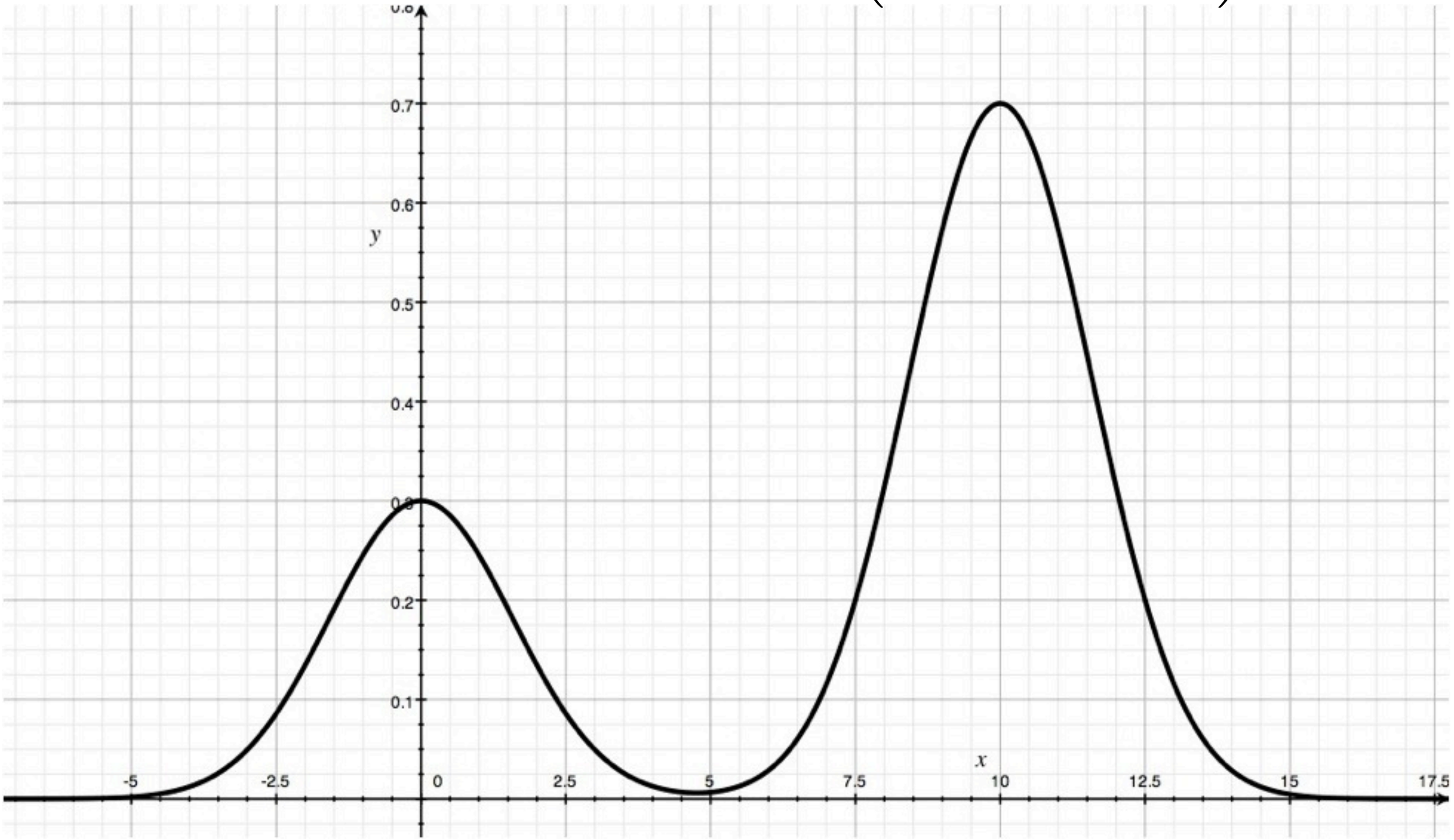
Number of steps

Action to take at each step

A third look

$$p(x) \propto 0.3 \exp(-0.2x^2) + 0.7 \exp(-0.2(x-10)^2)$$

$$0.3\exp\left(-0.2x^2\right) \;+\; 0.7\exp\left(-0.2\left(x-10\right)^2\right)$$



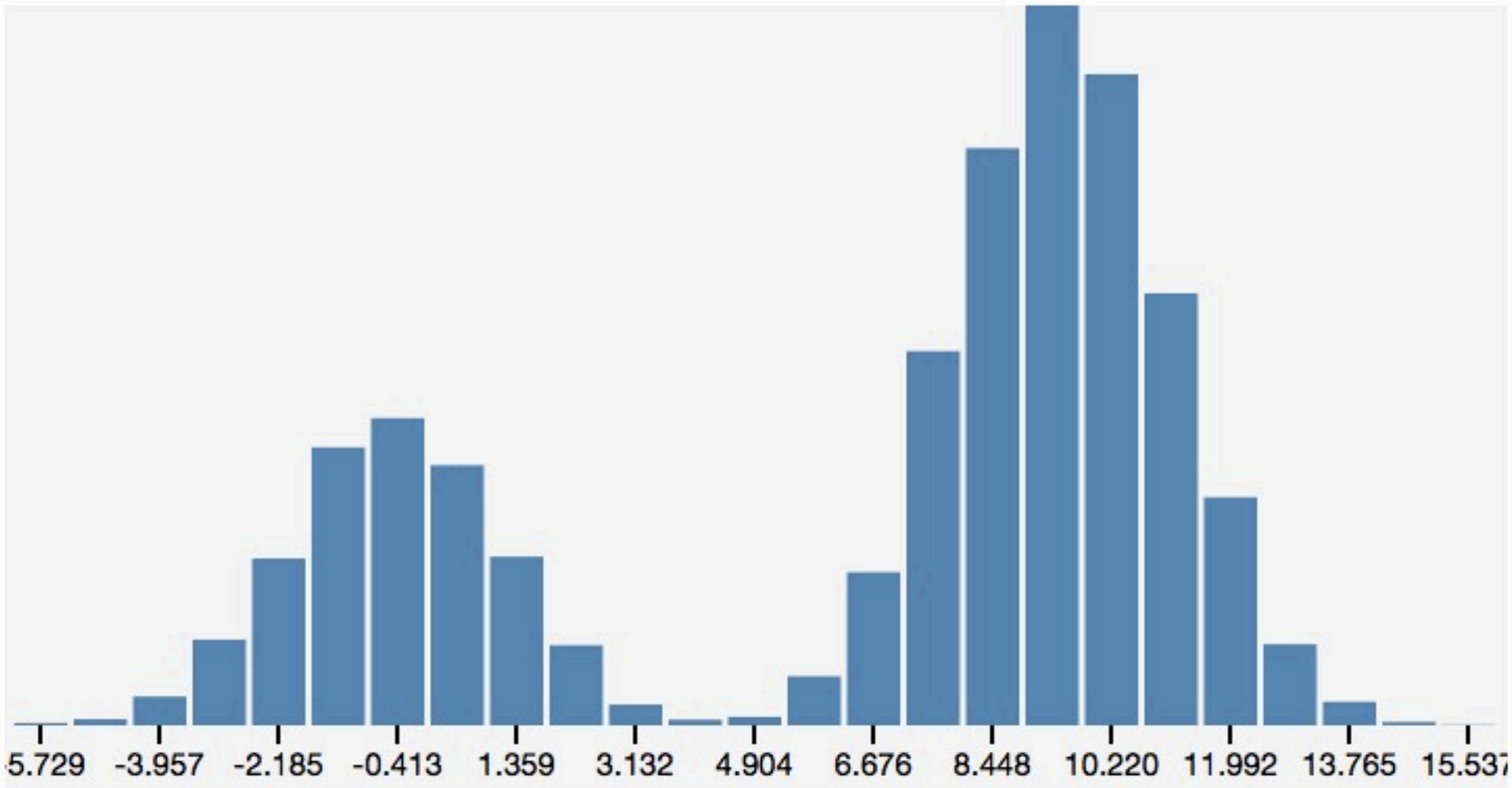
```haskell
mhTest :: IO ()
mhTest = do
    g ← MWC.createSystemRandom
    let (t,p) = (... , ...)
        mh = metropolisHastings t p
        act = every 100 (batchViz vizMH 50)
    walk mh [0] (10^6) g act
```

Number of samples plotted: 10000

x

Min: -5.7291215850716135

Max: 16.422778145776434



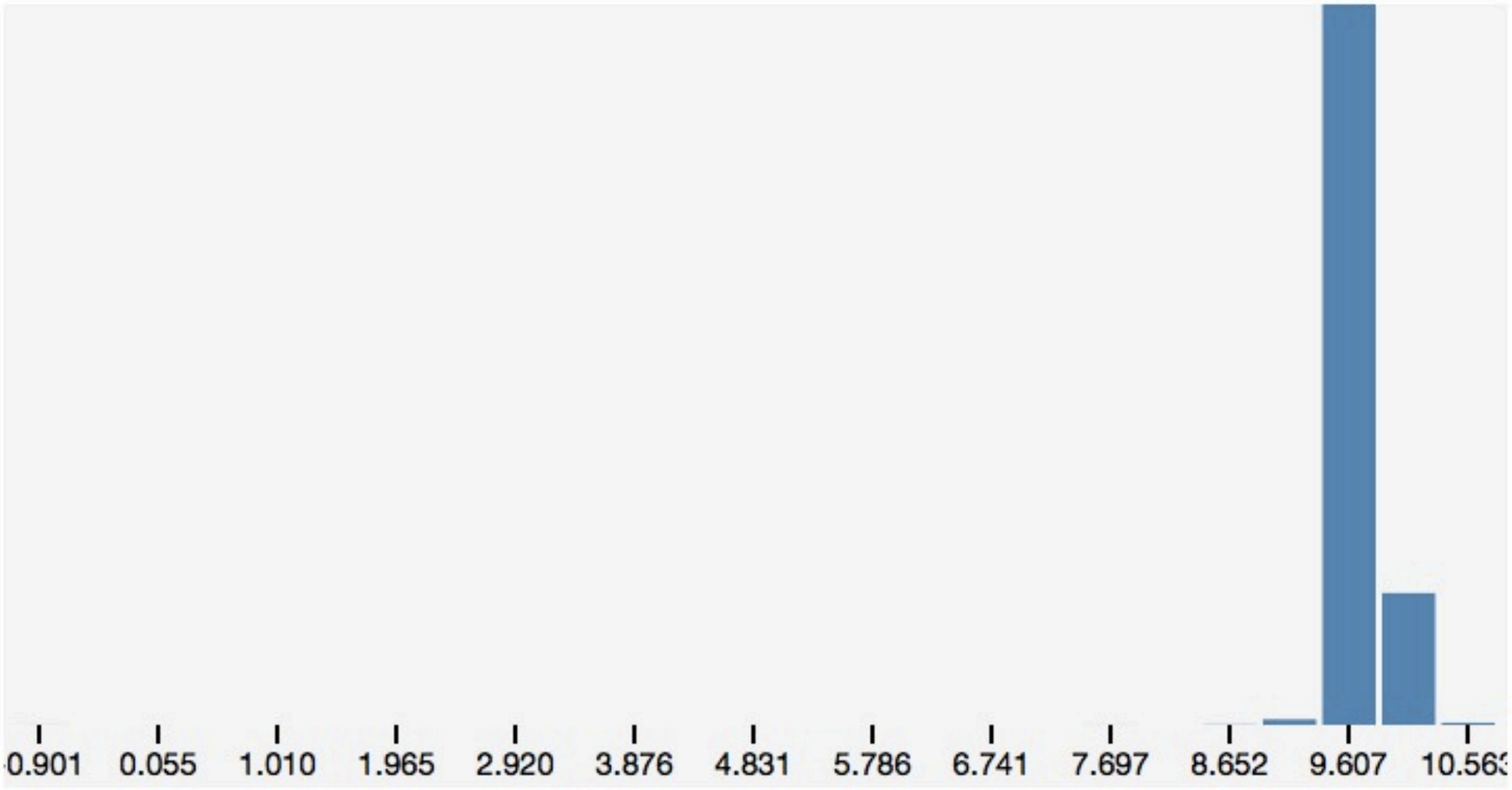
```haskell
saTest :: IO ()
saTest = do
    g ← MWC.createSystemRandom
    let (t,p) = (... , ...)
        sa = simulatedAnnealing t p
        coolSch temp = temp * (1 - 1e-3)
        x0 = ([0], 1, coolSch)
        act = every 100 (batchViz vizSA 50)
  walk sa x0 (10^6) g act
```

Number of samples plotted: 9889

x

Min: -0.9005223518480108

Max: 11.040135975148 39



```haskell
blockMH :: Step [Double]
blockMH = let target = normal [0,1,4,7] (diag [2,2,2,2])
              mh4D = metropolisHastings target
              mh1 = mh4D $ updateBlock 1 2
                           (λy → normal y (diag [1,1]))
          mh2 = mh4D $ updateBlock 3 3 (λy → normal y [[1]])
          mh3 = mh4D $ updateNth 4 (λy → normal y [[5]])
      in mixSteps [mh1, mh2, mh3] [0.5, 0.4, 0.7]
```

```haskell
blockTest :: IO ()
blockTest = do
  g ← MWC.createSystemRandom
  let a = batchPrint printMH 50
  walk blockMH [0,0,0,0] (10^6) g a
```

[0.907,-1.067,2.128,5.558],
[0.907,-1.067,2.128,6.584],
[1.336,-1.198,2.128,6.584],
[1.336,-1.198,2.128,6.584],
[-0.857,-1.729,2.128,6.584],
[-0.857,-1.729,2.660,6.584],
[-0.857,-1.729,2.660,6.584],
[-0.857,-1.729,2.660,8.325],
[-0.857,-1.729,2.660,8.325],
[-0.857,-1.729,2.660,8.325],
[0.598,-2.241,2.660,8.325],
[0.598,-2.241,2.660,8.325],
[0.598,-2.241,2.660,8.325],
[0.598,-2.241,2.660,7.699],
[0.598,-2.241,2.660,7.699], ...

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Kernel combinators for changing the accept ratio

More sampling methods

Approximate MH, Stochastic EM, Reversible jump MCMC

Thank you