Each question is worth 25 points. The assignment is due on April 24 in class. Show your work. Exercise set 10.2: 22, 24, 26, 34

22. Determine whether the graph is bipartite. You may find it useful to apply Theorem 4 and answer the question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.

\begin{tikzpicture}
    
    
    \node (x1) at (0,0) {X};
    \node (x2) at (1,1) {O};
    \node (x3) at (1,-1) {O};
    \node (x4) at (2,0) {X};
    \node (x5) at (0,-2) {X};
    \node (x6) at (2,2) {O};

    \draw (x1) -- (x2);
    \draw (x1) -- (x3);
    \draw (x1) -- (x4);
    \draw (x1) -- (x5);
    \draw (x2) -- (x6);
    \draw (x3) -- (x6);
    \draw (x4) -- (x6);
    \draw (x5) -- (x6);

\end{tikzpicture}

It follows by Theorem 4 that this graph is bipartite.

24. Determine whether the graph is bipartite. You may find it useful to apply Theorem 4 and answer the question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.

\begin{tikzpicture}
    
    \node (x1) at (0,0) {X};
    \node (x2) at (1,1) {O};
    \node (x3) at (1,-1) {O};
    \node (x4) at (2,0) {X};
    \node (x5) at (0,-2) {X};
    \node (x6) at (2,2) {O};

    \draw (x1) -- (x2);
    \draw (x1) -- (x3);
    \draw (x1) -- (x4);
    \draw (x1) -- (x5);
    \draw (x2) -- (x6);
    \draw (x3) -- (x6);
    \draw (x4) -- (x6);
    \draw (x5) -- (x6);

\end{tikzpicture}

It follows by Theorem 4 that this graph is bipartite.

26. For which values of $n$ are these graphs bipartite?

a) $K_n$: while $n = 2$, $K_n$ is bipartite
b) $C_n$: while $n$ is even, $K_n$ is bipartite
c) $W_n$: $W_n$ is not bipartite for any $n$
d) $Q_n$: $Q_n$ is bipartite for all values of $n$
34. Let n be a positive integer. Show that a subgraph induced by a nonempty subset of the vertex set of $K_n$ is a complete graph.

**Definition:** Complete Graphs: The complete graph with n graph vertices is denoted $K_n$ and has $\frac{n(n-1)}{2}$ edges.

For value $n = 2$, the subset of that graph is a complete graph. Assume that value $n = k$ is true, any value for $n = (k - 1)$ is also true.

Any subgraph of $K_n$ is $K_{(n-x)}$ where $x$ is an integer $> n$. 