CSCI 503B:
Sample Midterm

1. Let $f(x) = \omega(g(x))$. Give an example for f and g where $log(f(x)) = \omega(log(g(x))$. Give an example for f, g where $log(f(x)) = O(log(g(x))$. Is it possible to have $f, g$where $log(f(x)) = o(log(g(x))$? Prove.

2. Is $n! = O((n-1)!)$?

3. Solve the recurrences; assume $T(1) = T(2) = T(3) = 1$

$$T(n) = T(n/3) + \log n$$

$$T(n) = T(n-1) - T(n-2) + 1$$

4. Consider the 0-1 knapsack problem where all items have value equal to their weight. The knapsack is size n. Argue that the greedy algorithm that always takes the heaviest item that will into the backpack first is not optimal. Bonus: argue that the greedy algorithm always fills at least half of the backpack.

5. Consider the rod-cutting problem. A cut can be of size 1, 2, ... n. Argue that, if shorter cuts are always more expensive than longer cuts, there is a greedy optimal algorithm. What is it?

6. Consider Huffmann coding on an alphabet with frequencies $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ What does the Huffman tree look like? Give a set of frequencies such that the Huffman tree has maximum depth.

7. Give a quadratic time algorithm for computing the longest increasing subsequence of a string.

8. If I did not allow my rod to be cut shorter than 3 inches, how would you modify the rod-cutting algorithm?

9. Give an algorithm to compute the edit distance between two strings. (you can look up edit distance) What is the complexity?