CSCI 503B:
HOMEWORK 1
Each question is worth 20 points. Show your work.

1. Let \( f(n) = n \) and \( g(n) = n^{1+\sin(n)} \). Is \( f(n) = O(g(n)) \)? Is \( g(n) = O(f(n)) \)? Prove your answers.

2. Give an example of a function which is \( o(1) \). Just use the definition (or think intuitively), prove your answer.

3. You are told that \( f(n) = \Theta(g(n)) \). Is it necessarily true that \( 2f(n) = \Theta(2g(n)) \)? Argue your answer. Is it true that \( (f(n))^2 = \Theta((g(n))^2) \)? Argue.

4. Prove that if \( f(n) = O(g(n)) \) then \( f(n) + g(n) = O(g(n)) \). If \( f(n) = \Omega(g(n)) \) then is it true that \( f(n) - g(n) = \Omega(g(n)) \)? Prove.

5. Find what is wrong with this sentence: “the running time of this algorithm is at least \( O(n^2) \).” Analyze from a mathematical point of view what the content of this statement is.