
Assignment #3: More on NFAs, Minimum DFAs, Regular Expressions
Due: Monday, March 2nd, at the beginning of class

Note: Part of this homework asks questions about regular expressions. Please, read the relevant portion from Chapter 1 in Sipser, 2nd ed. We will discuss regular expressions during Thursday’s discussion on as-needed basis.

1. Sipser 2nd ed, 1.41 and 1.42.

2. Let \( \Sigma = \{a, b\} \).
   (a) Give an NFA \( N_1 \) for the regular expression \((a \cup ab)^*\).
   (b) Convert \( N_1 \) into an NFA \( N_2 \) without \( \epsilon \)-transitions.
   (c) Convert \( N_2 \) into a DFA.
   (d) Convert this DFA into a regular expression.
   (e) Prove that this regular expression is equivalent to \((a \cup ab)^*\).

3. Consider the DFA given by \( (Q = \{1, 2, 3, 4, 5, 6, 7\}, \{a, b\}, 1, \delta, F = \{3, 6\}) \) and \( \delta \) given by the following table.
   \[
   \begin{align*}
   \delta(1, a) &= 2 & \delta(1, b) &= 4 \\
   \delta(2, a) &= 3 & \delta(2, b) &= 2 \\
   \delta(3, a) &= 3 & \delta(3, b) &= 3 \\
   \delta(4, a) &= 7 & \delta(4, b) &= 5 \\
   \delta(5, a) &= 6 & \delta(5, b) &= 5 \\
   \delta(6, a) &= 6 & \delta(6, b) &= 6 \\
   \delta(7, a) &= 7 & \delta(7, b) &= 7
   \end{align*}
   
   Using the Minimization Algorithm discussed in class, construct a minimal DFA for the above DFA. Show your work.

4. Give regular expressions for the following languages given \( \Sigma = \{0, 1\} \):
   (a) \( \{w | w \text{ has length at least 3 and its third symbol is 0}\} \).
   (b) \( \{w | w \text{ contains at least two 0s and at most one } 1\} \).
   (c) \( \{w | w \text{ does not contain the string 101}\} \).

5. Sipser 2nd ed, 1.17, 1.19 and 1.20.