1. Exercise 0.3 from Sipser, 2nd ed.

2. Exercise 0.6 from Sipser, 2nd ed.

3. Let $p$ be a prime number. Prove that $\sqrt{p}$ is an irrational number.

4. Give a constructive proof to show that for all $n \geq 0$

$$\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2$$

Observe that $(n + 1)^4 - n^4 = 4n^3 + 6n^2 + 4n + 1$.

5. The sequence $a_n$ is defined recursively for $n \in \mathbb{N}$ by $a_1 = 3$, $a_2 = 5$ and

$$a_n = 3a_{n-1} - 2a_{n-2}$$

for $n \geq 3$. Prove that $a_n = 2^n + 1$ for every $n \in \mathbb{N}$.

6. Let’s define the set $B$ of binary trees as follows:

(a) A tree with a single root $r$ is in $B$.

(b) If $r$ is a node and $T_1 \in B$ and $T_2 \in B$ (i.e. $T_1$ and $T_2$ are binary trees), then the tree $T = (r, T_1, T_2) \in B$. $T$ is the tree with root $r$, and $r$ having $T_1$ as its left child and $T_2$ as its right child.

Define a node of a binary tree to be full if it has both a non-empty left and a non-empty right child. Prove by induction that the number of full nodes in a binary tree is 1 less than the number of its leaves.

7. Exercise 0.8 from Sipser, 2nd ed.

8. Exercise 0.9 from Sipser, 2nd ed.

9. Prove that for any undirected graph the number of odd-degree vertices is even, and the sum of the degrees of all vertices is even.