This quiz has 3 questions, for a total of 10 points.

1. **3 points** Apply the Partition algorithm to the following array. Write down the array after each step, drawing lines between the partitions as was done in the lecture and the textbook.

   
   \[2, 9, 2, 7\]

   **Solution:**
   
   \[
   \begin{align*}
   &\text{[| | 2, 9, 2 | 7]} \\
   &\text{[2 || 9, 2 | 7]} \quad \text{(1 point)} \\
   &\text{[2 | 9 | 2 | 7]} \quad \text{(1 point)} \\
   &\text{[2, 2 | 9 | 7]} \quad \text{(1 point)} \\
   &\text{[2, 2 | 7 | 9]} \\
   \end{align*}
   \]

2. **3 points** What are the loop invariants that we used to prove the correctness of the Partition algorithm? Recall that a loop invariant is a logical statement that is true at the beginning and end of each loop iteration.

   **Solution:** The loop invariants are
   
   - All the elements in the range \([0, i+1]\) are less or equal to the pivot. (1.5 points)
   - All the elements in the range \([i + 1, j]\) are greater than the pivot. (1.5 points)

3. **4 points** Write down an implementation of quicksort in Python. The parameters for quicksort are the same as those of partition but quicksort does not have a return value.

   **Solution:**
   
   ```python
   def quicksort(A, start, end):
       if start != end:
           # 1 point
           pivot = partition(A, start, end) # 1 point
           quicksort(A, start, pivot) # 1 point
           quicksort(A, pivot+1, end) # 1 point
   ```