Bounded-rational theory of mind for conversational implicature

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Logical Methods for Discourse
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Layers, stages

Continuations when?

▶ A: I’ll be Wild Bill.
   B: And I’ll be Calamity Jane.
   A: Look, Calamity Jane, I’ve found a gold nugget.
   B: We’re rich.
   A: Your dad is here now, so I guess you have to go.

▶ A: What kind of Scope does your mom use?
   B: What kind of soap?
   A: No, mouthwash; what kind of Scope?
   B: Oh, the regular kind.

▶ Bush complained about the ‘utterly [inaudible] loudspeakers’ in the room.

Alice | Bob | Carol

? ♂ ♀

Bob
WHAT IF I HAD SOME ICE CREAM? WOULDN'T THAT BE AWESOME?

GREAT, YOU'VE TRAPPED US IN A HYPOTHETICAL SITUATION!

NO, STOP—

MMM, ICE CREAM.

MAYBE IF I HAD A KNIFE I COULD CUT OUR WAY FREE...

MMM, ICE CREAM!

HERE, TAKE THIS ONE.
Game-theoretic pragmatics

Nature

Speaker

Hearer

Nature

(Solving online? . . . offline?)
Game-theoretic pragmatics

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Game

collaborative task

processing effort

Solution concept

perfect rationality

bounded rationality

Strategy

literal meaning

scalar implicature . . .

(Solving online? . . . offline?)
Game-theoretic pragmatics

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perfect rationality

literary meaning

risk of misinterpretation

bounded rationality

scalar implicature . . .

(Solving online? . . . offline?)
The good soldier Švejk
“The engine that you are to take off to the depot in Lysá nad Labem is no. 4268. Now pay careful attention. The first figure is four, the second is two, which means that you have to remember 42. That’s twice two. That means that in the order of the figures 4 comes first. 4 divided by 2 makes 2 and so again you’ve got next to each other 4 and 2. Now, don’t be afraid! What’s twice 4? 8, isn’t it? Well, then, get it into your head that 8 is the last in the series of figures in 4268. And now, when you’ve already got in your head that the first figure is 4, the second 2 and the fourth 8, all that’s to be done is to be clever and remember the 6 which comes before the 8. And that’s frightfully simple. The first figure is 4, the second is 2, and 4 and 2 are 6. So now you’ve got it: the second from the end is 6 and now we shall never forget the order of figures. You now have indelibly fixed in your mind the number 4268. But of course you can also reach the same result by an even simpler method . . .”
probabilistic model
(e.g., grammar)
approximate inference
(e.g., comprehension)

probabilistic model
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probabilistic model
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Grice and Marr

approximate inference
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probabilistic model
(e.g., grammar)
Probabilistic models invoke inference.
Random choices manipulate continuations.
Multiple layers track who thinks what.

- Probabilistic models
- Inference algorithms
- The hearer’s program
- The speaker’s program

We have a hammer. (Nails: anaphora? vagueness? …)
## Probabilistic models

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<td>$\overline{n}$</td>
<td>$\lambda c. \lambda g. c(0)(g) \cdot c(1)(g)$</td>
<td>fork server</td>
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\overline{A} \overset{\text{def}}{=} (A \to \text{assignment} \to \text{tree}) \to \text{assignment} \to \text{tree}
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<td>+</td>
<td>(n \rightarrow n \rightarrow n)</td>
<td>(\lambda x. \lambda y. x + y)</td>
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\(\bar{n}\) | \(\lambda c. \lambda g. c(0)(g) c(1)(g)\) | fork server     |
| +           | \(n \rightarrow n \rightarrow n\) | \(\lambda x. \lambda y. x + y\) | primitive       |
| flip + flip | \(\bar{n}\) | \(\lambda c. \lambda g. c(0)(g) c(1)(g) c(1)(g) c(2)(g)\) |               |
| Lower       | \(\bar{A} \rightarrow \text{tree } A\) | \(\lambda m. m(\lambda v. \lambda g. v)(\emptyset)\) | new thread      |

\[\bar{A} \overset{\text{def}}{=} (A \rightarrow \text{assignment} \rightarrow \text{tree}) \rightarrow \text{assignment} \rightarrow \text{tree}\]
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<td>Lower(flip + flip)</td>
<td>tree $n$</td>
<td>$0 1 1 2$</td>
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<td>tree $n \to n$</td>
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$$\bar{A} \overset{\text{def}}{=} (A \to \text{assignment} \to \text{tree}) \to \text{assignment} \to \text{tree}$$
## Perceptual observations

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<td>$x$</td>
<td>$\overline{n}$</td>
<td>$\lambda c. \lambda g. c(g(x))(g)$</td>
<td>get var</td>
</tr>
<tr>
<td>$x := \text{flip; }$</td>
<td>$\overline{A} \rightarrow \overline{A}$</td>
<td>$\lambda m. \lambda c. \lambda g. \begin{array}{c} \text{set var} \ m(c)(g[0/x]) \ m(c)(g[1/x]) \end{array}$</td>
<td>set var</td>
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<td>get var</td>
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| \( x := \text{flip}; \) | \( \bar{A} \rightarrow \bar{A} \) | \( \lambda m. \lambda c. \lambda g. \frac{50\%}{50\%} \)
\( m(c)(g[0/x]) \quad m(c)(g[1/x]) \) | set var |

**Lower**

\( (x := \text{flip}; \ y := \text{flip}; \)
\( \quad \text{if } x \lor y \text{ then } x \)
\( \quad \text{else fail}) \)

\( \text{tree } n \)

\[ \bar{A} \overset{\text{def}}{=} (A \rightarrow \text{assignment} \rightarrow \text{tree}) \rightarrow \text{assignment} \rightarrow \text{tree} \]
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<td>x := flip;</td>
<td>$\overline{A} \rightarrow \overline{A}$</td>
<td>$\lambda m. \lambda c. \lambda g. m(c)(g[0/x]) m(c)(g[1/x])$</td>
<td>set var</td>
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**Lower**

(x := flip; y := flip;
  if $x \lor y$ then x
  else fail)

**Lower**

(w := ...;
  if $w \models u$
  then $a := \text{act}; U(a|w)$
  else fail)

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More tractable inference

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<td>Lower</td>
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<td>tree n</td>
<td>lazy evaluation (branching heuristic)</td>
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<tr>
<td>(x := flip; y := flip; if x ∨ y then x else fail)</td>
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<td>( n )</td>
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<td>((x := \text{flip}; y := \text{flip};)</td>
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<td></td>
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<tr>
<td>\textbf{if} ( x \lor y ) \textbf{then} ( x )</td>
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<td>enumerate tree leaves</td>
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<td><strong>ApproxExpect</strong></td>
<td>tree</td>
<td>( n \rightarrow \overline{n} )</td>
<td>sample tree leaves</td>
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The bounded-rational hearer’s program

\begin{align*}
\text{ApproxExpect} \\
(\text{Lower(count := } 2 * \text{flip + flip;}) \\
\quad \text{conjunction := flip;} \\
\quad \text{if count,conjunction } \models \text{some,not_all} \\
\quad \text{then } a := \text{act; } \mathcal{U}(a \mid \text{count}) \\
\quad \text{else fail}))
\end{align*}
The bounded-rational hearer’s program

```plaintext
ApproxExpect
(Lower(count := 2 * flip + flip;
  conjunction := flip;
  if ((some ∧ not_all) → conjunction)
    ∧ (some → count > 0) ∧ (not_all → count < 3)
  then a := act; U(a | count)
  else fail))
```
The bounded-rational hearer’s program

ApproxExpect
(Lower(count := 2 * flip + flip;
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The bounded-rational hearer’s program

ApproxExpect
(\text{Lower}(\text{count} := 2 \times \text{flip} + \text{flip};
    \text{conjunction} := \text{flip};
    \text{if } ((\text{some} \land \text{not\_all}) \rightarrow \text{conjunction})
      \land (\text{some} \rightarrow \text{count} > 0) \land (\text{not\_all} \rightarrow \text{count} < 3)
      \text{then } a := \text{act}; U(a | \text{count})
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\text{ApproxExpect}
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\text{if } ((\text{some} \land \text{not_all}) \rightarrow \text{conjunction})
\]
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\land (\text{some} \rightarrow \text{count} > 0) \land (\text{not_all} \rightarrow \text{count} < 3)
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\[
\text{then } a := \text{act}; U(a | \text{count})
\]
\[
\text{else fail})
\]
Going meta

The hearer

▶ believes utterance is grammatical and true
  (constrains unobserved random variables)
▶ desires to maximize expected utility
▶ processes complex utterances less accurately because
  they trigger more constraints (e.g., ‘but’ deepens tree)

The speaker

▶ believes private world knowledge
▶ desires to maximize expected utility
▶ trades off informativity against complexity
  (e.g., omission, white lies)

The linguist

▶ invokes inference algorithms in probabilistic models
  (but can abstract; e.g., layperson model of meteorologist)
▶ programs in an intuitive and expressive language
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http://okmij.org/ftp/kakuritu/
http://okmij.org/ftp/kakuritu/incite.ml