Self-applicable probabilistic inference without interpretive overhead

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Patrick Hughes
Probabilistic inference

\[
\begin{align*}
\Pr(W) \\
\Pr(F|W) \\
\text{Observed evidence } F
\end{align*}
\]

Compute \( \Pr(W|F) \), etc.
Declarative probabilistic inference

<table>
<thead>
<tr>
<th>Model (what)</th>
<th>Inference (how)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pr(W)$</td>
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Declarative probabilistic inference

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<thead>
<tr>
<th>Toolkit (BNT)</th>
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<tbody>
<tr>
<td></td>
<td>invoke →</td>
<td>distributions,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>conditionalization,</td>
</tr>
<tr>
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<td></td>
<td>...</td>
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<tr>
<td>Language (BLOG)</td>
<td>random choice,</td>
<td>← interpret</td>
</tr>
<tr>
<td></td>
<td>evidence observation,</td>
<td></td>
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<td>BNT</td>
<td>use existing facilities: libraries, compilers,</td>
<td>add custom procedures: just sidestep or</td>
</tr>
<tr>
<td></td>
<td>types, debugging</td>
<td>extend</td>
</tr>
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<td>Language</td>
<td>succinct and natural: sampling procedures,</td>
<td>compile models to more efficient</td>
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<td>BLOG</td>
<td>relational programs</td>
<td>inference code</td>
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Declarative probabilistic inference
### Declarative probabilistic inference

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| Language (BLOG) | succinct and natural: sampling procedures, relational programs | compile models to more efficient inference code |

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<th>Today: best of both worlds</th>
<th>invoke →</th>
<th>← interpret</th>
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<tr>
<td>models of inference: theory of mind</td>
<td>deterministic parts of models run <em>at full speed</em></td>
<td></td>
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Express both models and inference as programs in the same general-purpose language.
Outline

▶ Expressivity (colored balls)
  Memoization

Inference (music)
  Reifying a model into a search tree
  Importance sampling with look-ahead

Self-interpretation (implicature)
  Variable elimination
  Particle filtering
  Theory of mind
Colored balls

An urn contains an unknown number of balls—say, a number chosen from a [uniform] distribution. Balls are equally likely to be blue or green. We draw some balls from the urn, observing the color of each and replacing it. We cannot tell two identically colored balls apart; furthermore, observed colors are wrong with probability 0.2. How many balls are in the urn? Was the same ball drawn twice? (Milch et al. 2007)
Colored balls

type color = Blue | Green

dist [(0.5, Blue); (0.5, Green)]
Colored balls

type color = Blue | Green

let ball_color = memo (function b ->
  dist [(0.5, Blue); (0.5, Green)])
Colored balls

type color = Blue | Green

let nballs = 1 + uniform 8 in
let ball_color = memo (function b ->
    dist [(0.5, Blue); (0.5, Green)])
Colored balls

type color = Blue | Green

let nballs = 1 + uniform 8 in
let ball_color = memo (function b ->
    dist [(0.5, Blue); (0.5, Green)]) in
let observe = function o ->
    if o <> observed_color(ball_color(uniform nballs)) then fail ()
Colored balls

type color = Blue | Green

let nballs = 1 + uniform 8 in
let ball_color = memo (function b ->
  dist [(0.5, Blue); (0.5, Green)]) in
let observe = function o ->
  if o <> observed_color(ball_color(uniform nballs))
  then fail ()
Colored balls

type color = Blue | Green
let opposite_color = function Blue -> Green
| Green -> Blue

let observed_color = function c ->
dist [(0.8, c); (0.2, opposite_color c)]

let nballs = 1 + uniform 8 in
let ball_color = memo (function b ->
dist [(0.5, Blue); (0.5, Green)]) in
let observe = function o ->
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let opposite_color = function Blue -> Green | Green -> Blue

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type color = Blue | Green
let opposite_color = function Blue -> Green
    | Green -> Blue

let observed_color = function c ->
    dist [(0.8, c); (0.2, opposite_color c)]

let model_nballs = function obs () ->
    let nballs = 1 + uniform 8 in
    let ball_color = memo (function b ->
        dist [(0.5, Blue); (0.5, Green)]) in
    let observe = function o ->
        if o <> observed_color (ball_color (uniform nballs))
        then fail () in
    Array.iter observe obs; nballs
Colored balls

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    then fail () in
Array.iter observe obs; nballs

normalize (sample_reify 17 10000 (model_nballs
    [|Blue;Blue;Blue;Blue;Blue;Blue;Blue;Blue;Blue;Blue;Blue;Blue;Blue|]))
Colored balls

```
type color = Blue | Green
let opposite_color = function Blue -> Green
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let observed_color = function c ->
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normalize (sample_reify 17 10000 (model_nballs [|Blue;Blue;Blue;Blue;Blue;Blue;Blue;Blue;Blue;Blue|]))
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  Reifying a model into a search tree
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Self-interpretation (implicature)
  Variable elimination
  Particle filtering
  Theory of mind
Reifying a model into a search tree

type 'a vc = V of 'a | C of (unit -> 'a pV)
and 'a pV = (float * 'a vc) list
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Depth-first traversal is exact inference by brute-force enumeration.
Reifying a model into a search tree

```
type 'a vc = V of 'a | C of (unit -> 'a pV)  
and  'a pV = (float * 'a vc) list
```

Inference procedures cannot access models’ source code.
Reifying a model into a search tree

\[
\begin{align*}
C & \xrightarrow{\text{reify}} \text{unit} \rightarrow \text{color} \\
& \xleftarrow{\text{reflect}} 
\end{align*}
\]

Implemented by representing (Filinski 1994) a state monad transformer (Moggi 1990) applied to a probability monad (Giry 1982) using \texttt{shift} and \texttt{reset} (Danvy & Filinski 1989) to operate on first-class delimited continuations (Felleisen et al. 1987) (Strachey & Wadsworth 1974)

- model runs inside \texttt{reset} (like an exception handler)
- \texttt{dist} and \texttt{fail} perform \texttt{shift} (like throwing an exception)
- \texttt{memo} mutates thread-local storage
Importance sampling with look-ahead

Probability mass $p_c = 1$
Importance sampling with look-ahead

1. Expand one level.

2. Report shallow successes.

3. Expand one more level and tally open probability.

4. Randomly choose a branch and go back to 2.

Probability mass $p_c = 1$
Importance sampling with look-ahead

1. Expand one level.
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Probability mass $p_c = 1$

(.2, Green)
Importance sampling with look-ahead

1. Expand one level.
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3. Expand one more level and tally open probability.

Probability mass $p_c = .75$

(0.2, Green)
Importance sampling with look-ahead

1. Expand one level.
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Probability mass $p_c = .75$

(.2, Green)
Importance sampling with look-ahead

1. Expand one level.
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Probability mass $p_c = 0.75$
(.2, Green) (.6, Blue)
Importance sampling with look-ahead

1. Expand one level.
2. Report shallow successes.
3. Expand one more level and tally open probability.
4. Randomly choose a branch and go back to 2.

Probability mass $p_c = 0$

$(.2, \text{Green}) (.6, \text{Blue})$
Importance sampling with look-ahead

1. Expand one level.
2. Report shallow successes.
3. Expand one more level and tally open probability.
4. Randomly choose a branch and go back to 2.

Probability mass $p_c = 0$
(.2, Green) (.6, Blue)
Music model

Pfeffer’s test of importance sampling (2007): motivic development in early Beethoven piano sonatas

Source motif $S$

- recursively divide
- Random binary tree

Destination motif $D$

- recursively concatenate
- Random binary tree

Want $Pr(D = \cdots | S = \cdots)$.

Exact inference and rejection sampling are infeasible.

Implemented using lists with stochastic parts.
Typical inference results

Pr(D = 1 | S = 1)

<table>
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<tr>
<th></th>
<th>IBAL</th>
<th>90 seconds</th>
<th>30 seconds</th>
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<tbody>
<tr>
<td>Mean</td>
<td>-14.6</td>
<td>-13.6</td>
<td>-13.7</td>
</tr>
<tr>
<td>SD</td>
<td>-15.1</td>
<td>-14.4</td>
<td>-13.8</td>
</tr>
<tr>
<td>#0</td>
<td>0</td>
<td>0</td>
<td>13</td>
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▶ **Self-interpretation (implicature)**
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  Theory of mind
Models of inference

Inference procedures and models

- are written in the same general-purpose language
- use the same stochastic primitive \texttt{dist}
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so inference procedures can be invoked by models

\begin{verbatim}
  inference (function () ->
    ... inference (function () -> ...) ...)
\end{verbatim}

and deterministic parts run at full speed.

Program generation with mutable state and control effects.
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- are written in the same general-purpose language
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and deterministic parts run at full speed.

Program generation with mutable state and control effects.

One common usage pattern: reify-infer-reflect

- Brute-force enumeration becomes \textit{variable elimination}
- Sampling becomes \textit{particle filtering}
Theory of mind

Instances abound:

- False-belief (Sally-Anne) task
- Trading securities
- Teacher’s hint to student
- Gricean reasoning in language use
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1. “Some professors are coming to the party.”
2. “All professors are coming to the party.”
3. “Some but not all professors are coming to the party.”

Trade-off between precision and ease of comprehension?
Theory of mind

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Trade-off between precision and ease of comprehension?

Crucial for collaboration among human and computer agents!

Want executable models.

A bounded-rational agent’s theory of bounded-rational mind
≈ approximate inference about approximate inference
Marr’s computational vs algorithmic models

\begin{align*}
\textbf{model} & \quad \Pr(W), \\
& \quad \Pr(\text{true}|W,F), \quad U(A,W) \\
\end{align*}

\begin{align*}
\text{world} & \quad W \in \{0 \text{ come}, 1 \text{ come}, 2 \text{ come}, 3 \text{ come}\} \times \cdots \\
\text{action} & \quad A \in \{\text{feed 0, feed 1, feed 2, feed 3}\} \\
\text{form} & \quad F \subseteq \{\text{some, all, no, not all}\}
\end{align*}
Marr’s computational vs *algorithmic* models

**model** \( \Pr(W), \ Pr(\text{true}|W,F), U(A,W) \)

**inference** \( \Pr(A|F, \text{true}) \)

- world \( W \in \{0 \text{ come}, 1 \text{ come}, 2 \text{ come}, 3 \text{ come}\} \times \cdots \)
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Marr’s computational vs algorithmic models

**model** $\Pr(W), U(A, W)$

**inference** $\Pr(F)$

**model** $\Pr(W), U(A, W)$

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**model** $\Pr(W)$,
$\Pr(\text{true}|W, F), U(A, W)$

**action** $\{\text{some, all, no, not all}\}$
Marr’s computational vs algorithmic models

inference $\Pr(F)$

model $\Pr(W), U(A, W)$

inference $\Pr(A|F, \text{true})$

model $\Pr(W)$,
$\Pr(\text{true}|W, F), U(A, W)$

A computational model of the modeler nests an algorithmic model of the modelee: invoke inference recursively, without interpretive overhead.
Summary

Express both models and inference as programs in the same general-purpose language.

- Combine strengths of toolkits and standalone languages
- Deterministic parts of models run at full speed
- Models can invoke inference without interpretive overhead
- Theory of mind: inference about approximate inference
- A variety of inference methods: variable elimination, particle filtering, importance sampling, ...?