Typed metaprogramming with effects

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LFMTP
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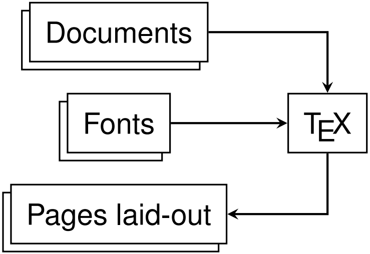
Accidental language design



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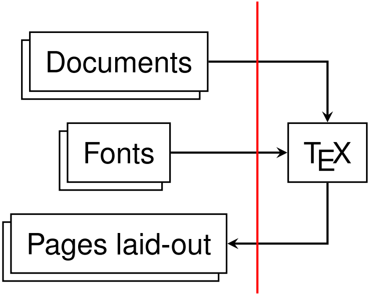

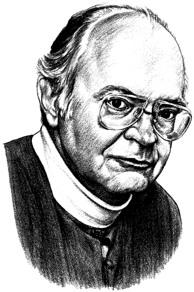

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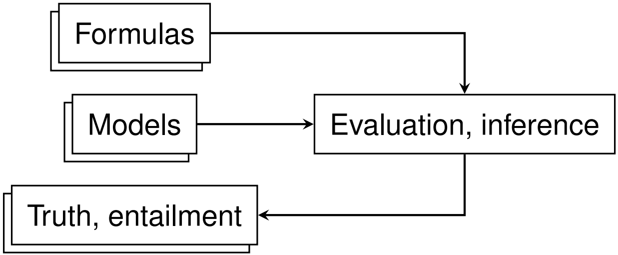

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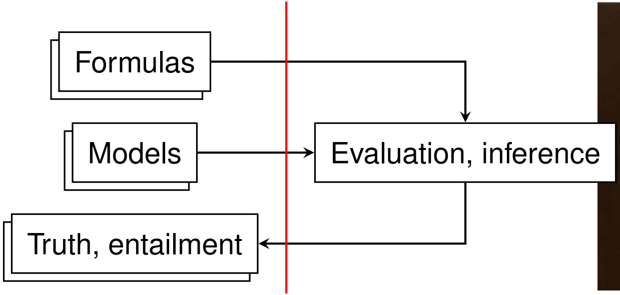
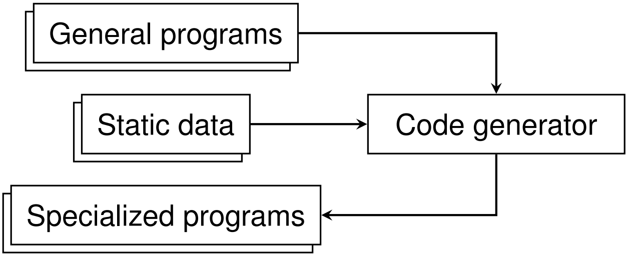


"It is probably more perspicuous to proceed indirectly, by

1. setting up a certain simple artificial language, that of tensed intensional logic,
2. giving the semantics of that language, and
3. interpreting English indirectly by showing in a rigorous way how to translate it into the artificial language.

This is the procedure we shall adopt . . . " —*Richard Montague*

Accidental language design



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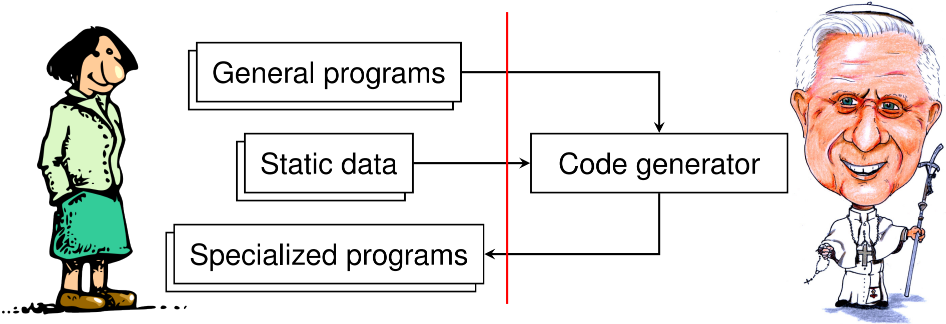


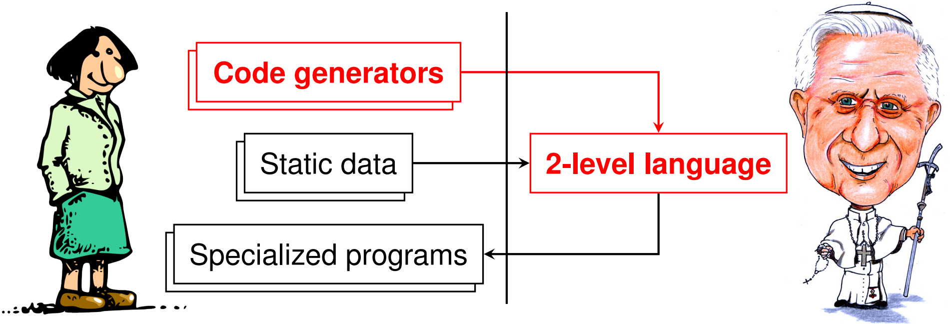
Optimizations specific to . . .

- Gaussian elimination
- Fast Fourier Transform
- Linear signal processing
- Embedded devices

Generate code using . . .

- Binding-time annotations
- Extensible compilers
- Side effects
- Custom generators

Accidental language design



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Generate code using . . .

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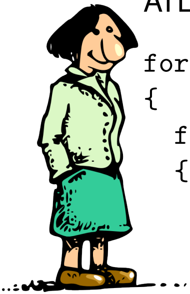
- Extensible compilers
- Side effects
- **Custom generators**

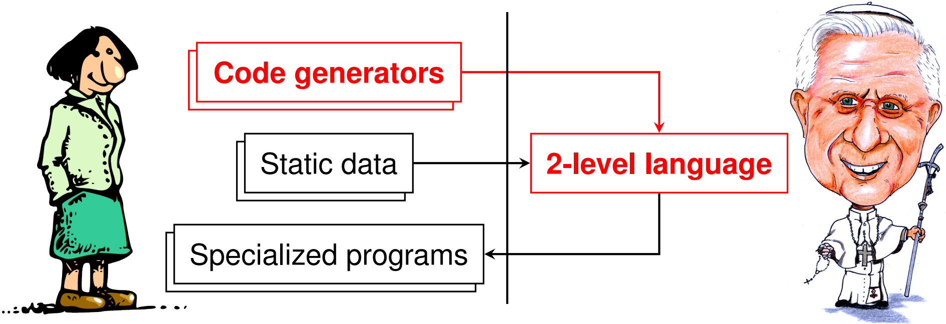
Accidental language design



ATLAS generates optimized code for matrix multiplication:

```
for (j=0; j < nu; j++)
{
  for (i=0; i < mu; i++)
  {
    if (Asg1stC && !k)
      fprintf(fpout, "%s  %s%d_%d = %s%d * %s%d;\n",
                  spc, rC, i, j, rA, i, rB, j);
    else
      fprintf(fpout, "%s  %s%d_%d += %s%d * %s%d;\n",
                  spc, rC, i, j, rA, i, rB, j);
    opfetch(fpout, spc, nfetch, rA, rB, pA, pB,
                mu, nu, offA, offB, lda, ldb, mulA, mulB,
                rowA, rowB, &ia, &ib);
  }
}
```

Accidental language design



Want **safety**: generate well-formed programs only
$\leftarrow$ track object variable bindings

Want **clarity**: generators resemble textbook algorithms
$\leftarrow$ provide delimited control operators

Outline

- **Delimited control for program generation**
  - Example
  - Formalization
- Natural-language semantics
  - Delimited control
  - Quotation
  - Variable binding
- Breaking the fourth wall
  - Contextual modalities
  - Environment classifiers

Gibonacci example

Like Fibonacci, but not always starting with 1 and 1.

```
let gib x y =
  let rec loop n =
    if n = 0 then x else
    if n = 1 then y else
    loop (n-1) + loop (n-2)
  in loop
```

`gib 1 1 5` $\longrightarrow$ `8`

Other domains:

- Gaussian elimination
- Fast Fourier Transform
- Linear signal processing
- Embedded devices …

Gibonacci example, specialized

Familiar from quasiquotation, macros, PE, or just `printf`.

```
let gib x y =
  let rec loop n =
    if n = 0 then x else
    if n = 1 then y else
    .<.~(loop (n-1)) + .~(loop (n-2))>.
  in loop
```

```
.<fun x y -> .~(gib .<x>. .<y>. 5)>.
⟶  .<fun x_0 -> fun y_1 ->
      ((((y_1 + x_0) + y_1) + (y_1 + x_0)) +
      ((y_1 + x_0) + y_1))>.
```

Code values can be open when evaluating under generated λ,
but the generated code is always well-scoped.
Binding context follows evaluation context, implicitly!

Gibonacci example, memoized

Keep a memo table as mutable state.

```
let gib x y = let memo = new_memo () in
  let rec loop n =
    if n = 0 then x else
    if n = 1 then y else
    memo loop (n-1) + memo loop (n-2)
  in loop
```

gib 1 1 5 $\longrightarrow$ 8

Other domain-specific optimizations:

- Dynamic programming
- Pivoting matrices
- Simplifying arithmetic on complex roots of unity ...

Gibonacci example, specialized, memoized?

A naive combination duplicates code, as when unfolding in PE.

```
let gib x y = let memo = new_memo () in
  let rec loop n =
    if n = 0 then x else
    if n = 1 then y else
    .<.~(memo loop (n-1)) + .~(memo loop (n-2))>.
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.<fun x y -> .~(gib .<x>. .<y>. 5)>.
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     ((y_1 + x_0) + y_1))>.
```

Generating code fast is not generating fast code!

Two problems

1. Code in state voids safety, due to **scope extrusion**.

```
let r = ref .<1>. in
.<fun y -> .~(r := .<y>.; .<()>.)>.;
!r
⟶ .<y_1>.
```

2. Need to **insert let** at top, not to duplicate specialized code.

```
.<fun x y -> .~(gib .<x>. .<y>. 4)>.
⟶ .<fun x_0 -> fun y_1 ->
      let t_2 = y_1 + x_0 in
      let t_3 = t_2 + y_1 in t_3 + t_2>.
```

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let r = ref .<1>. in
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```

loop 2 (→ `let t_2 = y_1 + x_0 in`, `t_2`)

loop 3 (→ `let t_3 = t_2 + y_1 in`, `t_3`)

(Similar: need to insert if/assert.)

Two solutions

1. Use CPS or monadic style to write the generator. (Match compiler, CPS translator (Danvy & Filinski), PE (Bondorf))

```
let gib x y =
  let rec loop n k =
    if n = 0 then k x else
    if n = 1 then k y else
    memo loop (n-1) (fun r1 ->
    memo loop (n-2) (fun r2 ->
    k .<.~r1 + .~r2>.))
  in loop
```

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```

loop 2 k *table* ≈ `.<let t_2 = y_1 + x_0 in .~(k .<t_2>. ` *table*′`)>.`

loop 3 k *table*′ ≈ `.<let t_3 = t_2 + y_1 in .~(k .<t_3>. ` *table*″`)>.`

Importing `k` under `let` is ok because code is opaque!

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```

```
.<fun x y -> .~(top_fn (gib .<x>. .<y>. 5))>.
⟶ .<fun x_0 -> fun y_1 ->
     let t_1 = y_1 in let t_0 = x_0 in
     let t_2 = t_1 + t_0 in
     let t_3 = t_2 + t_1 in
     let t_4 = t_3 + t_2 in t_4 + t_3>.
```

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```

Two solutions

2. Use *delimited control operators* to hide CPS.
   (CPS translator (Danvy & Filinski), PE (Lawall & Danvy))

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let gib x y =
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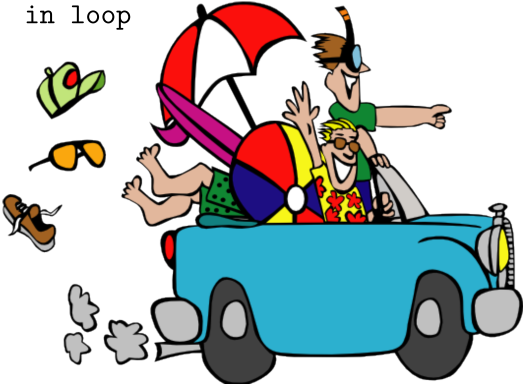

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```

$\langle D[$`loop 2`$]\rangle$ *table* $\approx$ `.<let t_2 = y_1 + x_0 in`
`.~(`$\langle D[$`.<t_2>.`$]\rangle$ *table*$'$`)>.`

$\langle D[$`loop 3`$]\rangle$ *table*$'$ $\approx$ `.<let t_3 = t_2 + y_1 in`
`.~(`$\langle D[$`.<t_3>.`$]\rangle$ *table*$''$`)>.`

Two solutions

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  in loop

.<fun x y -> .~(top_fn (fun () -> gib .<x>. .<y>. 5))>.
⟶ .<fun x_0 -> fun y_1 ->
     let t_1 = y_1 in let t_0 = x_0 in
     let t_2 = t_1 + t_0 in
     let t_3 = t_2 + t_1 in
     let t_4 = t_3 + t_2 in t_4 + t_3>.
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  in loop

top_fn (fun () -> .<fun x y -> .~(gib .<x>. .<y>. 5)>.)
⟶ .<let t_1 = y_1 in let t_0 = x_0 in
      let t_2 = t_1 + t_0 in
      let t_3 = t_2 + t_1 in
      let t_4 = t_3 + t_2 in
      fun x_0 -> fun y_1 -> t_4 + t_3>.
```

Preventing scope extrusion

Custom generators $\neq$ Fixed generator

Low-hanging fruit:

For safety, simply **treat later binders as earlier delimiters** in the operational semantics and type system.

(Existing practice; Thiemann & Dussart's constraint on state)

Our source language $\lambda_1^{\varnothing}$

Expressions $e ::= x \mid i \mid e + e \mid \lambda x.\, e \mid \mathsf{fix} \mid e e$
$\mid (e, e) \mid \mathsf{fst} \mid \mathsf{snd} \mid \mathsf{ifz}\ e\ \mathsf{then}\ e\ \mathsf{else}\ e$
$\mid$ 出 $\mid \{e\} \mid \langle e \rangle \mid {\sim}e$

$$C[(\lambda x.\, e)\, v] \rightsquigarrow C[e[x := v]] \qquad (\beta_v)$$

$$\vdots$$

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Expressions $e ::= x \mid i \mid e + e \mid \lambda x.\, e \mid \mathsf{fix} \mid ee$
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Delimited control (Felleisen, . . . , Danvy & Filinski)

Code generation (Davies & Pfenning, . . . , Taha)

$$C\big[(\lambda x.\, e)\, v\big] \rightsquigarrow C\big[e[x := v]\big] \qquad (\beta_v)$$

$$\vdots$$

Staging

Two levels: present 0, future 1.

Expressions $e ::= x \mid i \mid e + e \mid \lambda x.\, e \mid \mathsf{fix} \mid ee$
$\mid (e, e) \mid \mathsf{fst} \mid \mathsf{snd} \mid \mathsf{ifz}\ e\ \mathsf{then}\ e\ \mathsf{else}\ e$
$\mid$ 出 $\mid \{e\} \mid \langle e \rangle \mid \sim e$

Delimited control (Felleisen, . . . , Danvy & Filinski)

Code generation (Davies & Pfenning, . . . , Taha)

$$C[(\lambda x.\, e)\, v] \rightsquigarrow C[e[x := v]] \qquad (\beta_v)$$

$$\vdots$$

Staging

Two levels: present 0, future 1.

$$
\begin{array}{llcl}
\text{Values} & v^0 & ::= & x \mid \lambda x.\, e \mid \langle v^1 \rangle \mid \cdots \\
 & v^1 & ::= & x \mid \lambda x.\, v^1 \mid v^1 v^1 \mid \cdots
\end{array}
$$

$$
\begin{array}{llcl}
\text{Contexts} & C^0 & ::= & C^0[\Box e] \mid C^0[v^0 \Box] \mid C^1[{\sim}\Box] \mid \Box \mid \cdots \\
 & C^1 & ::= & C^1[\Box e] \mid C^1[v^1 \Box] \mid C^0[\langle \Box \rangle] \mid \cdots
\end{array}
$$

$$
\begin{aligned}
C^0[(\lambda x.\, e)\, v^0] &\rightsquigarrow C^0[e[x := v^0]] && (\beta_v) \\
C^1[{\sim}\langle v^1 \rangle] &\rightsquigarrow C^1[v^1] && ({\sim}) \\
&\vdots
\end{aligned}
$$

Staging

Two levels: present 0, future 1.

$$
\begin{aligned}
\text{let } f = \lambda x.\, x \text{ in } \langle \lambda t.\, {\sim}(f \langle t \rangle) \rangle &\leadsto_{\beta_v} \langle \lambda t.\, {\sim}((\lambda x.\, x)\langle t \rangle) \rangle \\
&\leadsto_{\beta_v} \langle \lambda t.\, {\sim}\langle t \rangle \rangle \\
&\leadsto_{\sim} \langle \lambda t.\, t \rangle
\end{aligned}
$$

$$
\begin{aligned}
C^0[(\lambda x.\, e)\, v^0] &\leadsto C^0[e[x := v^0]] && (\beta_v) \\
C^1[{\sim}\langle v^1 \rangle] &\leadsto C^1[v^1] && ({\sim}) \\
&\vdots
\end{aligned}
$$

Control

Two operators: shift 出, reset { }.

$$\text{Expressions} \quad e ::= x \mid i \mid e+e \mid \lambda x.\, e \mid \mathsf{fix} \mid ee$$
$$\mid (e,e) \mid \mathsf{fst} \mid \mathsf{snd} \mid \mathsf{ifz}\ e\ \mathsf{then}\ e\ \mathsf{else}\ e$$
$$\mid \underbrace{出 \mid \{e\}}_{\text{Delimited control}} \mid \underbrace{\langle e \rangle \mid {\sim} e}_{\text{Code generation}}$$

Delimited control (Felleisen, . . . , Danvy & Filinski)

Code generation (Davies & Pfenning, . . . , Taha)

$$C^0[(\lambda x.\, e)\, v^0] \rightsquigarrow C^0[e[x := v^0]] \qquad (\beta_v)$$
$$C^1[{\sim}\langle v^1 \rangle] \rightsquigarrow C^1[v^1] \qquad ({\sim})$$
$$C^0[\{v^0\}] \rightsquigarrow C^0[v^0] \qquad (\{\})$$
$$C^0[\{D[出 v^0]\}] \rightsquigarrow C^0[\{v^0(\lambda x.\, \{D[x]\})\}] \qquad (出^0)$$
$$\vdots$$

Control

Two operators: shift 出, reset { }.

$$\begin{aligned} \{1+1\}+1 &\rightsquigarrow_{+} \{2\}+1 \\ &\rightsquigarrow_{\{\}} 2+1 \\ &\rightsquigarrow_{+} 3 \end{aligned}$$

$$\begin{aligned} C^0[(\lambda x.\, e)\, v^0] &\rightsquigarrow C^0[e[x := v^0]] && (\beta_v) \\ C^1[\sim\langle v^1\rangle] &\rightsquigarrow C^1[v^1] && (\sim) \\ C^0[\{v^0\}] &\rightsquigarrow C^0[v^0] && (\{\}) \\ C^0[\{D[\text{出} v^0]\}] &\rightsquigarrow C^0[\{v^0(\lambda x.\, \{D[x]\})\}] && (\text{出}^0) \\ &\vdots \end{aligned}$$

Control

Two operators: shift 出, reset { }. Emulate state (Filinski).

$$\mathsf{const} = \lambda y.\lambda z.y \quad \mathsf{get} = 出(\lambda k.\lambda z.kzz) \quad \mathsf{put} = \lambda z'.出(\lambda k.\lambda z.kz'z')$$

$$\begin{aligned}
\{\mathsf{const}\,(\mathsf{get} + 40)\}\,2 &\rightsquigarrow_{出^0} \{(\lambda k.\,\lambda z.\,kzz)(\lambda x.\,\{\mathsf{const}(x+40)\})\}\,2 \\
&\rightsquigarrow_{\beta_v} \{\lambda z.\,(\lambda x.\,\{\mathsf{const}(x+40)\})zz\}\,2 \\
&\rightsquigarrow_{\{\}} (\lambda z.\,(\lambda x.\,\{\mathsf{const}(x+40)\})zz)\,2 \\
&\rightsquigarrow_{\beta_v} (\lambda x.\,\{\mathsf{const}(x+40)\})\,2\,2 \\
&\rightsquigarrow_{\beta_v} \{\mathsf{const}(2+40)\}\,2 \rightsquigarrow_{\beta_v} \{\lambda z.\,42\}\,2 \rightsquigarrow^{+} 42
\end{aligned}$$

$$\begin{aligned}
C^0[(\lambda x.\,e)\,v^0] &\rightsquigarrow C^0[e[x := v^0]] && (\beta_v) \\
C^1[\sim\langle v^1\rangle] &\rightsquigarrow C^1[v^1] && (\sim) \\
C^0[\{v^0\}] &\rightsquigarrow C^0[v^0] && (\{\}) \\
C^0[\{D[出 v^0]\}] &\rightsquigarrow C^0[\{v^0(\lambda x.\,\{D[x]\})\}] && (出^0) \\
&\vdots
\end{aligned}$$

Control

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$$\mathsf{const} = \lambda y.\lambda z.y \quad \mathsf{get} = 出(\lambda k.\lambda z.kzz) \quad \mathsf{put} = \lambda z'.出(\lambda k.\lambda z.kz'z')$$

$$\begin{aligned}
\{\mathsf{const}\ (\mathsf{put}(\mathsf{get}+1)+\mathsf{get})\}\ 2 &\rightsquigarrow^+ \{\mathsf{const}\ (\mathsf{put}(2+1)+\mathsf{get})\}\ 2 \\
&\rightsquigarrow_+ \{\mathsf{const}\ (\mathsf{put}\ 3+\mathsf{get})\}\ 2 \\
&\rightsquigarrow^+ (\lambda x.\,\{\mathsf{const}(x+\mathsf{get})\})\ 3\ 3 \\
&\rightsquigarrow_{\beta_v} \{\mathsf{const}(3+\mathsf{get})\}\ 3 \\
&\rightsquigarrow^+ \{\mathsf{const}(3+3)\}\ 3 \rightsquigarrow^+ 6
\end{aligned}$$

$$\begin{aligned}
C^0[(\lambda x.\,e)\,v^0] &\rightsquigarrow C^0[e[x := v^0]] && (\beta_v) \\
C^1[\sim\langle v^1\rangle] &\rightsquigarrow C^1[v^1] && (\sim) \\
C^0[\{v^0\}] &\rightsquigarrow C^0[v^0] && (\{\}) \\
C^0[\{D[出 v^0]\}] &\rightsquigarrow C^0[\{v^0(\lambda x.\,\{D[x]\})\}] && (出^0) \\
&\vdots
\end{aligned}$$

Staging + Control

Is scope extrusion possible?

$$\{\text{const } (\text{let } x = \langle \lambda y.\, {\sim}(\text{put } \langle y \rangle) \rangle \text{ in get})\}\ \langle 0 \rangle$$

$$\rightsquigarrow^{+}\ \{\text{const } (\text{let } x = \langle \lambda y.\, {\sim}(\langle y \rangle) \rangle \text{ in get})\}\ \langle y \rangle$$

$$\rightsquigarrow^{+}\ \{\text{const get}\}\ \langle y \rangle$$

$$\rightsquigarrow^{+}\ \{\text{const } \langle y \rangle\}\ \langle y \rangle$$

$$\rightsquigarrow^{+}\ \langle y \rangle$$

Staging + Control

Is scope extrusion possible? No. Level-1 λ delimits control.

$$\{\mathsf{const}\ (\mathsf{let}\ x = \langle \lambda y.\, {\sim}(\mathsf{put}\,\langle y \rangle)\rangle\ \mathsf{in}\ \mathsf{get})\}\ \langle 0 \rangle$$

$$\leadsto^{+}\ \{\mathsf{const}\ (\mathsf{let}\ x = \langle \lambda y.\, {\sim}\{(\lambda k.\, \lambda z.\, k\langle y \rangle \langle y \rangle)(\lambda x.\, \{\langle {\sim} x \rangle\})\}\rangle\ \mathsf{in}\ \mathsf{get})\}\ \langle 0 \rangle$$

Staging + Control

Is scope extrusion possible? No. Level-1 λ delimits control.

$$\{\text{const}\ (\text{let}\ x = \langle \lambda y.\, {\sim}(\text{put}\ \langle y \rangle) \rangle\ \text{in get})\}\ \langle 0 \rangle$$

$$\rightsquigarrow^{+} \{\text{const}\ (\text{let}\ x = \langle \lambda y.\, {\sim}\{(\lambda k.\, \lambda z.\, k\langle y \rangle\langle y \rangle)(\lambda x.\, \{\langle {\sim}x \rangle\})\} \rangle\ \text{in get})\}\ \langle 0 \rangle$$

Can write:

- memoizing fixpoint, CPS translation, partial evaluation
- dynamic programming
- Gaussian elimination
- Markov models with ‘symbolic’ matrix multiplications

Cannot write:

- loop-invariant code motion
- inserting `let`/`if`/`assert` at outermost possible scope

$$\{\langle \lambda i.\, {\sim}(\text{出} \lambda k.\, \langle \text{let}\ x = 40 + 2\ \text{in}\ {\sim}(k\langle i + x \rangle) \rangle) \rangle\}$$

Outline

Formal linguistics

Goal: relate forms to meanings in a concise specification.
Science, rather than engineering.

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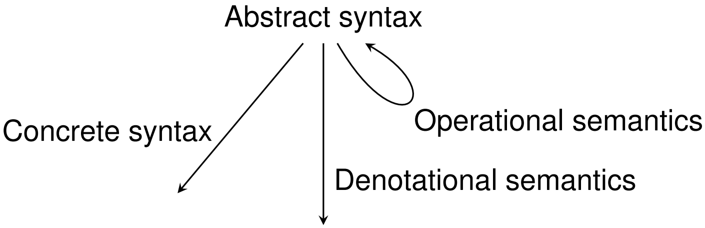
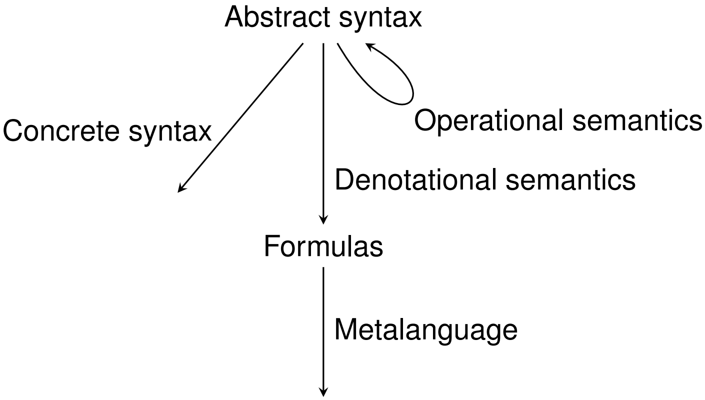
Goal: relate forms to meanings in a concise specification.
Science, rather than engineering.



Formal linguistics

Goal: relate forms to meanings in a concise specification.
Science, rather than engineering.



Cf. introductory logic.

Truth and entailment

Alice left and Bob left.

$\downarrow$

$left\ a \wedge left\ b$

Truth and entailment

$$
\begin{array}{ccc}
\text{Alice left and Bob left.} & \longmapsfrom & \text{Alice left.} \\
\downarrow & & \downarrow \\
\mathit{left}\ a \wedge \mathit{left}\ b & \longmapsfrom & \mathit{left}\ a
\end{array}
$$

Truth and entailment

Alice left and Bob left. ⟻ Alice left.

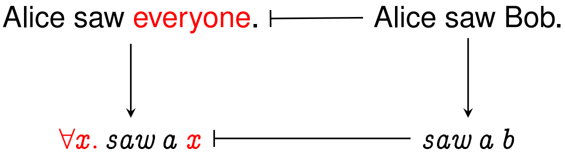
↓ ↓

$\mathit{left}\ a \wedge \mathit{left}\ b \longmapsfrom \mathit{left}\ a$

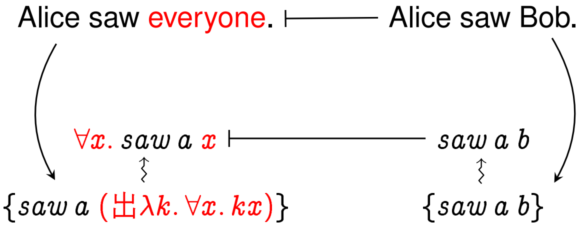
Alice and Bob left. ⟻ Alice left.

Alice and Bob met. ⟻̸ Alice met.

Delimited control for quantifiers



Delimited control for quantifiers



Delimited control for quantifiers

Alice saw everyone. ⟼ Alice saw Bob.

$\forall x.\, saw\; a\; x \longmapsfrom saw\; a\; b$

$\{saw\; a\; (出 \lambda k.\, \forall x.\, kx)\}$ $\qquad$ $\{saw\; a\; b\}$

Everyone saw someone.

$\forall x.\, \exists y.\, saw\; x\; y$

$\{saw\; (出 \lambda k.\, \forall x.\, kx)\; (出 \lambda k.\, \exists y.\, ky)\}$

Delimited control for quantifiers

Alice saw everyone. $\longmapsfrom$ Alice saw Bob.

$$\forall x.\ saw\ a\ x \longmapsfrom saw\ a\ b$$

$$\{saw\ a\ (出\lambda k.\ \forall x.\ kx)\} \rightsquigarrow \forall x.\ saw\ a\ x \qquad \{saw\ a\ b\} \rightsquigarrow saw\ a\ b$$

Everyone saw someone.

$$\forall x.\ \exists y.\ saw\ x\ y$$

$$\{saw\ (出\lambda k.\ \forall x.\ kx)\ (出\lambda k.\ \exists y.\ ky)\} \rightsquigarrow \forall x.\ \exists y.\ saw\ x\ y$$

Simulate other **linguistic side effects**: pronouns, questions, . . .

Evaluation order

Surface scope is preferred over **inverse scope**:

- Everyone saw someone.

Anaphora is preferred over **cataphora**:

- Everyone's father saw her mother.
  $*$ Her father saw everyone's mother.

Gap tends to precede **wh-phrase**:

- Who do you think saw what?
  $*$ What do you think who saw?

Reuse the same default of left-to-right evaluation for a more concise explanation.

Outline

Varieties of quotation

'Bachelor' has eight letters.

$\downarrow$ pure

$\textit{has-8-letters}$ 'bachelor'

Quine says 'quotation has a certain anomalous feature'.

$\downarrow$ direct

$\textit{say } q$ ⟨quotation has a certain anomalous feature⟩

Varieties of quotation

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$\textit{say}\ q\ \langle\texttt{quotation has a certain anomalous feature}\rangle$

Quine says quotation has a certain anomalous feature.

$\downarrow$ indirect

$\textit{say}\ q\ (\textit{has-a-certain-anomalous-feature quotation})$

Varieties of quotation

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Quine says quotation has a certain anomalous feature.

$\downarrow$ indirect

$\textit{say}\ q\ (\textit{has-a-certain-anomalous-feature quotation})$

Quine says quotation 'has a certain anomalous feature'.

$\downarrow$ mixed

$\textit{say}\ q$ (⟨has a certain anomalous feature⟩ $\textit{quotation}$) ???

Mixing mention and use

Quine says quotation ‘has a certain anomalous feature’.

↓ mixed

$\ldots$ (eval q ⟨has a certain anomalous feature⟩) $\ldots$

Bush is proud of his ‘eckullectic’ reading list.

↓ mixed

$\ldots$ (eval b ⟨eckullectic⟩) $\ldots$

Mixing mention and use

Quine says quotation ‘has a certain anomalous feature’.

↓ mixed

…(eval q ⟨has a certain anomalous feature⟩)…

Bush is proud of his ‘eckullectic’ reading list.

↓ mixed

…(eval b ⟨eckullectic⟩)…

Yet Cheney’s reading list is far more ‘eckullectic’, not to mention longer.

↓ mixed

…(eval b ⟨eckullectic⟩)…

Program generation in natural language

Bush boasted of ‘my [Cheney’s favorite adjective] reading list’.

$\downarrow$ syntactic unquotation

$$\ldots \sim(\mathit{favorite\ adjective}\ c) \ldots$$

Bush boasted of ‘my [eclectic] reading list’.

$\downarrow$ semantic unquotation

$$\ldots \sim(出\lambda k.\, \exists x.\, \mathtt{eval}\ b\ x = \mathit{eclectic} \wedge k x) \ldots$$

Program generation in natural language

Bush boasted of ‘my [Cheney’s favorite adjective] reading list’.

↓ syntactic unquotation

$$\ldots \sim(\mathit{favorite\ adjective\ c}) \ldots$$

Bush boasted of ‘my [eclectic] reading list’.

↓ semantic unquotation

$$\ldots \sim(\text{出}\lambda k.\,\exists x.\,\mathsf{eval}\ b\ x = \mathit{eclectic} \wedge kx) \ldots$$

Bush complained about the ‘utterly [inaudible] loudspeakers’ in the room.

$$\ldots \sim(\text{出}\lambda k.\,\exists x.\,\mathit{inaudible}\ x \wedge kx) \ldots$$

$$\ldots \sim(\text{出}\lambda k.\,\exists x.\,\mathsf{eval}\ b\ x = \mathit{inaudible} \wedge kx) \ldots$$

Variable binding in natural language

The teacher praised every boy who did his homework.

$$\downarrow$$

$$\ldots (出 \lambda k.\, \forall x.\, (\mathit{boy}\ x \wedge \mathit{did}\ x\ (\mathit{homework}\ x)) \to k x) \ldots$$

The teacher praised ‘every boy who did [his homework]’.

$$\downarrow$$

$$\ldots \mathtt{eval}\ t\ \langle 出 \lambda k.\, \forall x.\, (\mathit{boy}\ x \wedge \mathit{did}\ x \sim\!\ldots) \to k x\rangle \ldots$$

Inverse quantifier scope

It is an attractive scientific hypothesis that evaluation order is *always* from left to right.

Everyone saw someone.

$$\{saw\,(\text{出}\lambda k.\,\forall x.\,kx)\,(\text{出}\lambda k.\,\exists y.\,ky)\} \rightsquigarrow \forall x.\,\exists y.\,saw\;x\;y$$

$$\{\langle\{saw\,(\text{出}\lambda k.\,\forall x.\,kx)\,{\sim}(\text{出}\lambda k.\,\langle\exists y.\,{\sim}(ky)\rangle)\}\rangle\}$$

$$\rightsquigarrow \langle\exists y.\,saw\,(\text{出}\lambda k.\,\forall x.\,kx)\;y\rangle$$

However, reducing generated shift statically is hard.

$$\{\langle\forall x.\,saw\;x\,{\sim}(\text{出}\lambda k.\,\langle\exists y.\,{\sim}(ky)\rangle)\rangle\}$$

If only . . .

Outline

Delimited control for program generation

- Example
- Formalization

Natural-language semantics

- Delimited control
- Quotation
- Variable binding

▶ **Breaking the fourth wall**

- Contextual modalities
- Environment classifiers

Loop-invariant code motion

We want:

$$\langle \lambda i.\, \text{let } y = i + (\ldots 40 + 2 \ldots) \text{ in } y + y \rangle \rightsquigarrow$$
$$\langle \text{let } x = 40 + 2 \text{ in } \lambda i.\, \text{let } y = i + x \text{ in } y + y \rangle$$

Or in a two-level calculus:

$$\underline{\lambda} i.\, (\underline{\lambda} y.\, y + y)(i + (\ldots \underline{40} + \underline{2} \ldots)) \rightsquigarrow$$

Or in CPS:

$$\begin{aligned} &(\underline{\lambda} i.\, \lambda k.\, (\underline{\lambda} y.\, \lambda k'.\, k'(y + y))(\lambda l. \\ &\qquad (\ldots \underline{40} + \underline{2} \ldots)(\lambda x. \\ &\qquad k(l(i + x))))) \\ &\quad (\lambda z.\, z) \rightsquigarrow \end{aligned}$$

Contextual modalities

In a two-level calculus:

$$\underline{\lambda} i.\, (\underline{\lambda} y.\, y + y) \\ (i + (\ldots \underline{40} + \underline{2} \ldots))$$

Manage environment explicitly using de Bruijn indices:

$$\underline{\lambda}((\underline{\lambda}(\mathtt{zero} + \mathtt{zero})) \\ (\mathtt{zero} + \text{出} \lambda k.\, (\underline{\lambda}(\mathtt{throw}\ (\mathtt{import}\ k)\ \mathtt{zero}))(\underline{40} + \underline{2})))$$

The continuation $k : [i : \mathsf{int}]\, \mathsf{int} \to []\, \mathsf{int}$

$\mathtt{import}\ k : [i : \mathsf{int}, x : \mathsf{int}]\, \mathsf{int} \to [x : \mathsf{int}]\, \mathsf{int}$

(Nanevski, Pfenning & Pientka 2008)

MetaOCaml today!

Environment classifiers

Judgments: $e : \tau \qquad \alpha / \tau_0 \qquad \alpha \leq \beta$

$$\frac{\alpha/\tau_0}{\underline{n} : \langle \mathsf{int} \rangle^\alpha} \qquad \frac{e_1 : \langle \mathsf{int} \rangle^\alpha \quad e_2 : \langle \mathsf{int} \rangle^\alpha}{e_1 + e_2 : \langle \mathsf{int} \rangle^\alpha}$$

$$\frac{e_1 : \langle v_1 \to v \rangle^\alpha \quad e_2 : \langle v_1 \rangle^\alpha}{e_1 e_2 : \langle v \rangle^\alpha} \qquad \frac{\begin{array}{c}[x : \langle v_1 \rangle^\alpha] \\ \vdots \\ e : (\langle v \rangle^\alpha \to \tau_0) \to \tau_0\end{array} \quad \alpha/\tau_0}{\underline{\lambda} x.\, e : (\langle v_1 \to v \rangle^\alpha \to \tau_0) \to \tau_0}$$

$$\frac{}{0/\mathsf{String}} \qquad \frac{\begin{array}{c}[\alpha \leq \beta \quad \beta/\tau_0] \\ \vdots \\ e : (\langle v \rangle^\beta \to \tau_0) \to \tau_0\end{array}}{\mathsf{region}\ e : (\langle v \rangle^\alpha \to \tau_0) \to \tau_0} \qquad \frac{e : \langle \tau \rangle^\alpha \quad \alpha \leq \beta}{e : \langle \tau \rangle^\beta}$$

Environment classifiers

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Environment classifiers

Judgments: $e : \tau$ $\quad \alpha / \tau_0$ $\quad \alpha \leq \beta$

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$$\frac{e_1 : \langle \upsilon_1 \to \upsilon \rangle^\alpha \quad e_2 : \langle \upsilon_1 \rangle^\alpha}{e_1 e_2 : \langle \upsilon \rangle^\alpha} \qquad \frac{\begin{array}{c}[x : \langle \upsilon_1 \rangle^\alpha] \\ \vdots \\ e : (\langle \upsilon \rangle^\alpha \to \tau_0) \to \tau_0 \quad \alpha / \tau_0\end{array}}{\underline{\lambda} x.\, e : (\langle \upsilon_1 \to \upsilon \rangle^\alpha \to \tau_0) \to \tau_0}$$

$$\frac{}{0 / \mathsf{String}} \qquad \frac{\begin{array}{c}[\alpha \leq \beta \quad \beta / \tau_0] \\ \vdots \\ e : (\langle \upsilon \rangle^\beta \to \tau_0) \to \tau_0\end{array}}{\mathsf{region}\ e : (\langle \upsilon \rangle^\alpha \to \tau_0) \to \tau_0} \qquad \frac{e : \langle \tau \rangle^\alpha \quad \alpha \leq \beta}{e : \langle \tau \rangle^\beta}$$

Using environment classifiers

In CPS:

$$(\underline{\lambda} i.\, \lambda k.\, (\underline{\lambda} y.\, \lambda k'.\, k'(y+y))(\lambda l.\, (\ldots \underline{40} + \underline{2} \ldots)(\lambda x.\, k(l(i+x)))))\,(\lambda z.\, z)$$

Create a region for $\underline{\lambda} i$.

$$\text{region } (\underline{\lambda} i.\, \lambda k.\, (\underline{\lambda} y.\, \lambda k'.\, k'(y+y))(\lambda l.\, (\lambda k.\, \lambda m.\, \lambda n.\, (\underline{\lambda} x.\, kxm)(\lambda l.\, n(l(\underline{40}+\underline{2}))))(\lambda x.\, k(l(i+x)))))\,(\lambda z.\, \lambda k.\, kz)(\lambda z.\, \lambda k.\, kz)$$

Continuation hierarchy: k up to $\underline{\lambda} i$. m up to $\underline{\lambda} x$. n beyond

OCaml today!

The ends

Metalanguages for

- high-performance/embedded computing
- natural-language semantics of scope and quotation

need

- **safety** $\leftarrow$ track object variable bindings
- **clarity** $\leftarrow$ provide delimited control operators

Help!