Grafting Trees: Continuations in Type Logical Grammar

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$$\begin{tikzcd}
\text{Computational} \arrow[r, bend right] & \text{Linguistics} \arrow[l, bend right]
\end{tikzcd}$$

What a linguist cares about

Entailment

Nobody liked any course	$\not\dashv \; \vdash$	Nobody liked any computer science course
Somebody liked a course	$\dashv \; \nvdash$	Somebody liked a computer science course

Truth conditions are part of sentence meanings.

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Ambiguity

Nobody liked a course. Nobody liked any course.

"linear scope" $\neg\exists x.\, \exists y.\, \mathsf{liked}(x, y)$ $\exists y.\, \neg\exists x.\, \mathsf{liked}(x, y)$ "inverse scope"

"You can fool some of the people all of the time, and all of the people some of the time, but you can not fool all of the people all of the time."

—Abraham Lincoln

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Acceptability

No student liked any course	*Every student liked any course
Few students liked any course	*Any student liked no course

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This talk deals with English, but the approach hopefully extends to other languages.

The Curry-Howard isomorphism

“Syntax”:

Deduction rules for proving grammaticality

“Semantics”:

Translation to a logical metalanguage

Acceptability

Ambiguity

Entailment

Outline

OLD	Multiplicative linear logic *for* linguistics	Alice liked Bob.
NEW	Delimited continuations *for* quantification	Alice liked everybody's mother.
OLD	Unary modalities *for* polarity sensitivity	*Alice liked anybody's mother. Nobody liked anybody's mother.
NEW	Evaluation order *for* linear precedence	*Anybody liked nobody's mother.
NEW	Staging *for* scope ambiguities	Somebody liked everybody's mother.

Payoffs

For linguistics:

- Cover more empirical data.
- Relate (denotational) semantics to (operational) psycholinguistics?

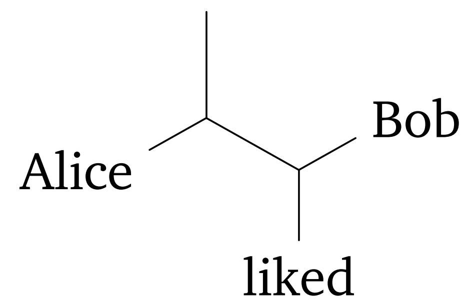
For computer science:

- Understand delimited continuations geometrically and logically.
- Staging side effects?

Linear logic for linguistics

Alice liked Bob.   *Alice liked.   *Alice liked Bob NJPLS.   *Alice Bob liked.

$$\dfrac{\text{Alice} \vdash np \qquad \dfrac{\text{liked} \vdash (np\backslash s)/np \quad \text{Bob} \vdash np}{\text{liked} \circ \text{Bob} \vdash np\backslash s}\,/\text{E}}{\text{Alice} \circ (\text{liked} \circ \text{Bob}) \vdash s}\,\backslash\text{E}$$



A sequent is *complete* iff its antecedent is built up using the $\circ$ connective only and its conclusion is s ("sentence").

Linear logic for linguistics

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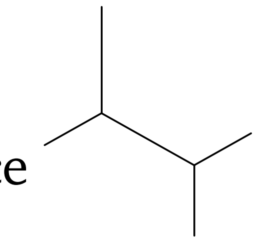


A sequent is *complete* iff its antecedent is built up using the $\circ$ connective only and its conclusion is s (“sentence”).

Natural-deduction rules for *categorial grammar* (a kind of *type logical grammar*)

$$\dfrac{\Gamma \vdash B \quad \Delta \vdash C}{\Gamma \circ \Delta \vdash B \circ C}\,\circ I \qquad \dfrac{\Delta \vdash B \circ C \quad \Gamma\{B \circ C\} \vdash A}{\Gamma\{\Delta\} \vdash A}\,\circ E \qquad \dfrac{}{A \vdash A}\,\text{Axiom}$$

$$\dfrac{\Delta \circ C \vdash A}{\Delta \vdash A/C}\,/I \qquad \dfrac{\Delta \vdash A/C \quad \Gamma \vdash C}{\Delta \circ \Gamma \vdash A}\,/E$$

$$\dfrac{B \circ \Delta \vdash A}{\Delta \vdash B\backslash A}\,\backslash I \qquad \dfrac{\Gamma \vdash B \quad \Delta \vdash B\backslash A}{\Gamma \circ \Delta \vdash A}\,\backslash E$$

Tensor

$B \circ C$ *means* “B followed by C” (non-commutative; non-associative)

Implications

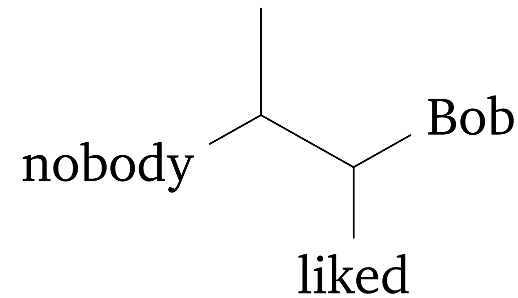
A/C *means* “makes A before C”

$B\backslash A$ *means* “makes A after B”

In-situ quantification

Everybody liked Bob. Somebody liked Bob. Nobody liked Bob.

$$\dfrac{\text{nobody} \vdash s/(np\backslash s) \qquad \dfrac{\text{liked} \vdash (np\backslash s)/np \quad \text{Bob} \vdash np}{\text{liked} \circ \text{Bob} \vdash np\backslash s}\,/\mathrm{E}}{\text{nobody} \circ (\text{liked} \circ \text{Bob}) \vdash s}\,/\mathrm{E}$$



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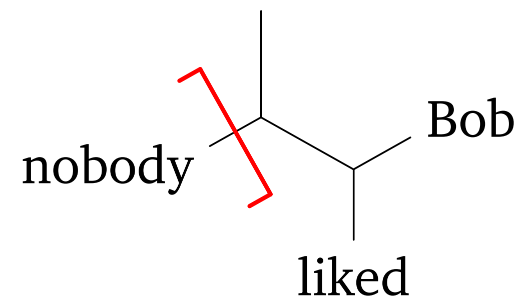
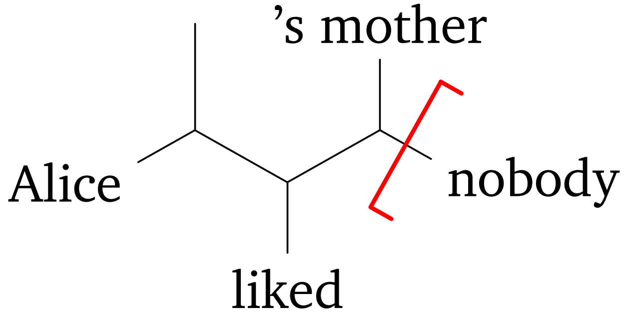
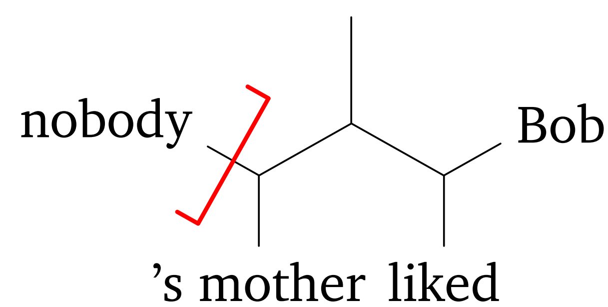


Alice liked [nobody]'s mother.

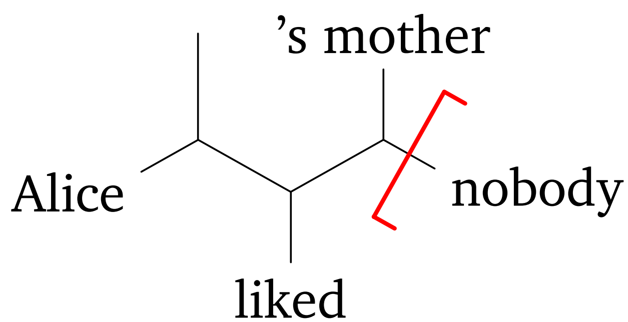


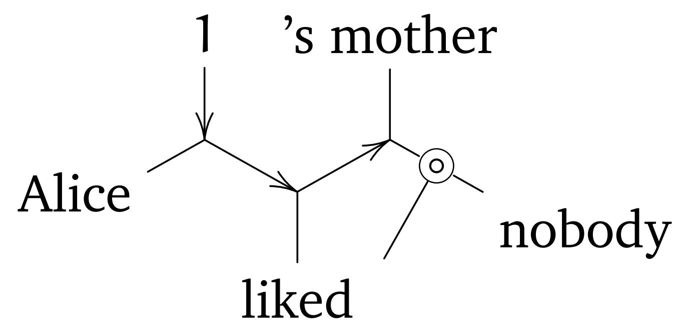
[Nobody]'s mother liked Bob.



Delimited continuations

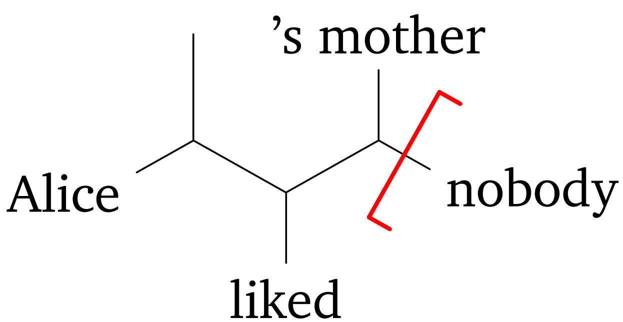
$\text{Alice} \circ \big(\text{liked} \circ (\text{nobody} \circ \text{'s mother})\big)$ $\quad$ $\text{nobody} \circledcirc \big(\text{'s mother} \triangleleft ((1 \triangleright \text{Alice}) \triangleright \text{liked})\big)$



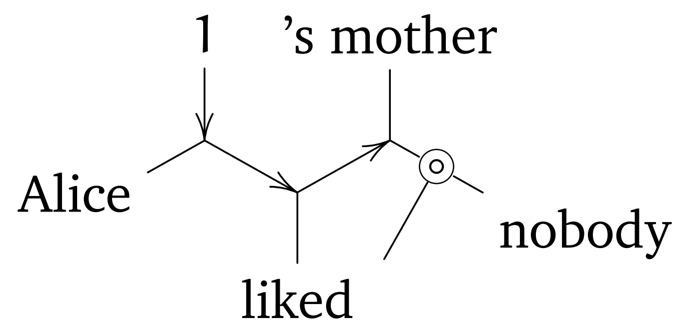


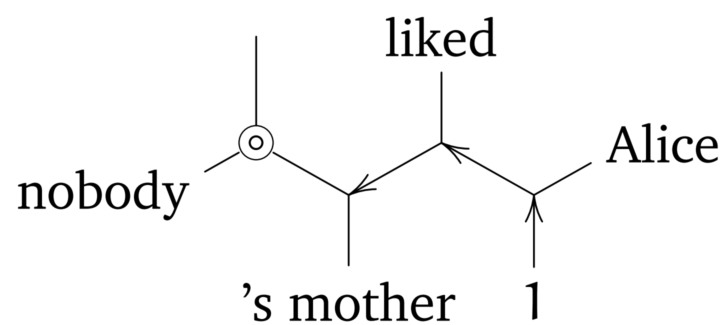
Delimited continuations

Alice $\circ$ (liked $\circ$ (nobody $\circ$'s mother))



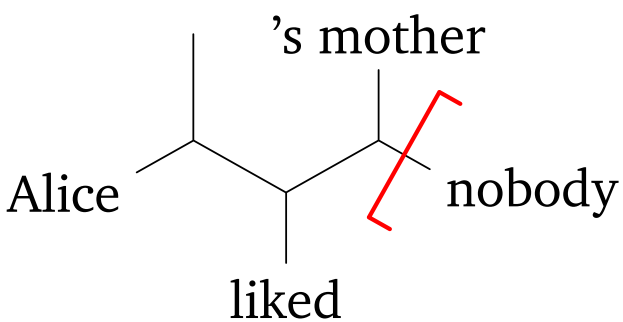
nobody $\circledcirc$ ('s mother $\triangleleft$ ((1 $\triangleright$ Alice) $\triangleright$ liked))

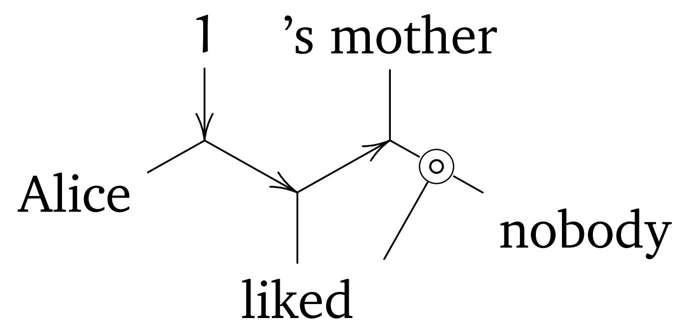




Delimited continuations

Alice $\circ$ (liked $\circ$ (nobody $\circ$'s mother))   nobody $\circledcirc$ ('s mother $\triangleleft$ ((1 $\triangleright$ Alice) $\triangleright$ liked))





Additional natural-deduction rules for *multimodal* categorial grammar

$$\frac{\Gamma \vdash B \quad \Delta \vdash C}{\Gamma \circledcirc \Delta \vdash B \circledcirc C}\circledcirc\mathrm{I} \qquad \frac{\Delta \vdash B \circledcirc C \quad \Gamma\{B \circledcirc C\} \vdash A}{\Gamma\{\Delta\} \vdash A}\circledcirc\mathrm{E}$$

$$\frac{\Delta \circledcirc C \vdash A}{\Delta \vdash A /\!\!/ C} /\!\!/\mathrm{I} \qquad \frac{\Delta \vdash A /\!\!/ C \quad \Gamma \vdash C}{\Delta \circledcirc \Gamma \vdash A} /\!\!/\mathrm{E}$$

$$\frac{B \circledcirc \Delta \vdash A}{\Delta \vdash B \backslash\!\!\backslash A} \backslash\!\!\backslash\mathrm{I} \qquad \frac{\Gamma \vdash B \quad \Delta \vdash B \backslash\!\!\backslash A}{\Gamma \circledcirc \Delta \vdash A} \backslash\!\!\backslash\mathrm{E}$$

Tensor

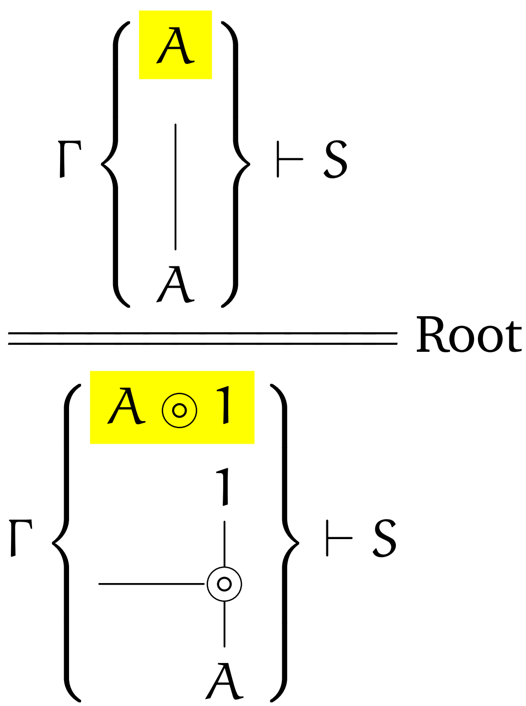
$B \circledcirc C$ *means* "B plugged into C"
(non-commutative; non-associative)

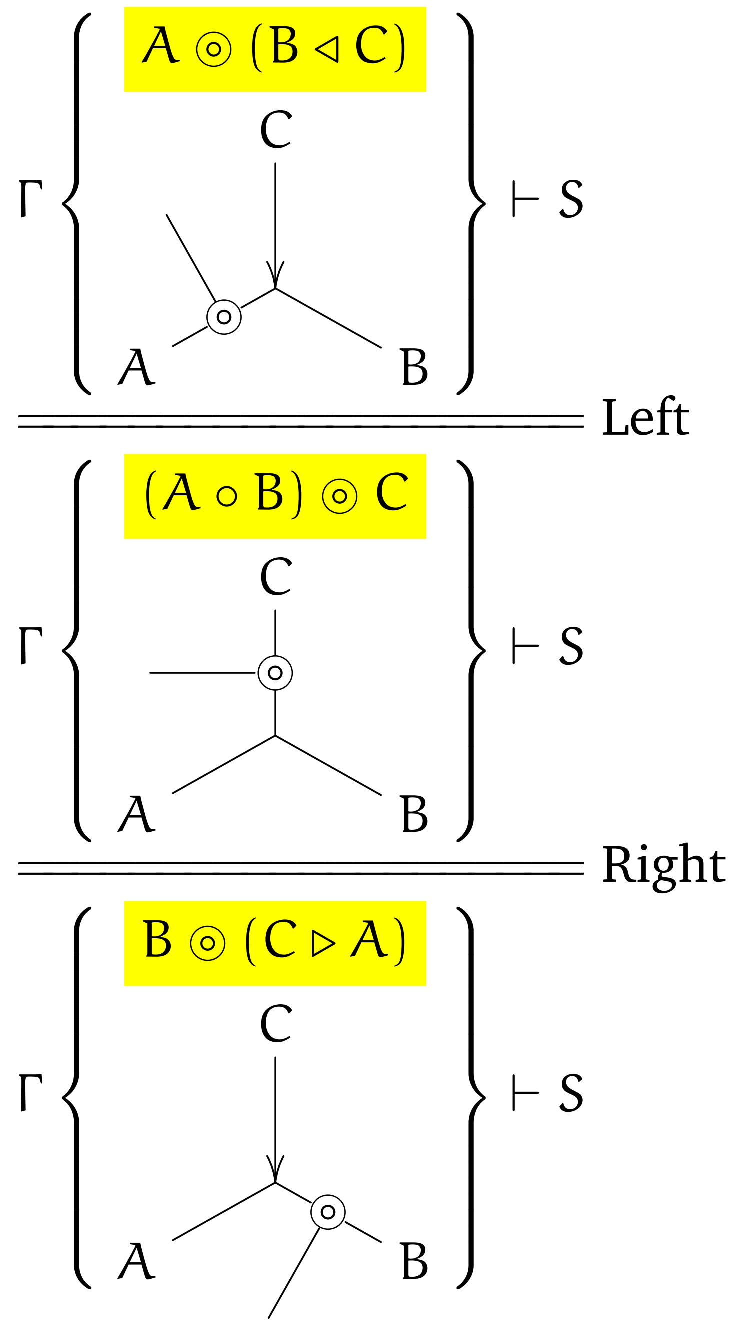
Implications

$A /\!\!/ C$ *means* "makes A inside C"
$B \backslash\!\!\backslash A$ *means* "makes A outside B"

Coming up: nobody $\vdash s /\!\!/ (np \backslash\!\!\backslash s)$

Delimited continuations: structural postulates





Delimited continuations: structural postulates

$$\frac{\Gamma\{A\} \vdash S}{\Gamma\{A \circledcirc 1\} \vdash S}\ \text{Root}$$

$$\begin{array}{cl} \Gamma\{A \circledcirc (B \triangleleft C)\} \vdash S & \\ \hline\hline \Gamma\{(A \circ B) \circledcirc C\} \vdash S & \text{Left} \\ \hline\hline \Gamma\{B \circledcirc (C \triangleright A)\} \vdash S & \text{Right} \end{array}$$

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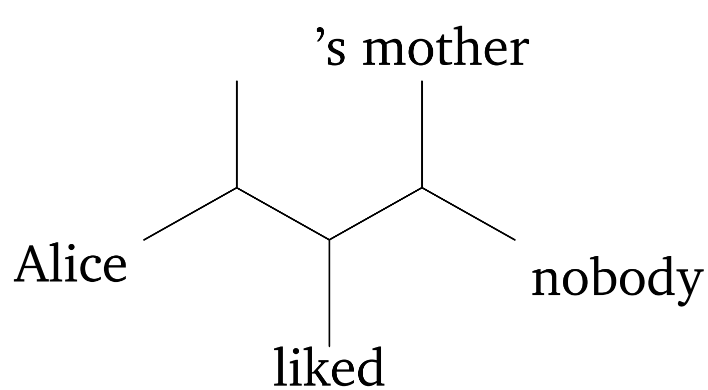
$$\begin{array}{c} \Gamma\{\text{Alice} \circ (\text{liked} \circ (\text{nobody} \circ \text{'s mother}))\} \vdash S \\ \hline\hline \Gamma\{(\text{Alice} \circ (\text{liked} \circ (\text{nobody} \circ \text{'s mother}))) \circledcirc 1\} \vdash S \\ \hline\hline \Gamma\{(\text{liked} \circ (\text{nobody} \circ \text{'s mother})) \circledcirc (1 \triangleright \text{Alice})\} \vdash S \\ \hline\hline \Gamma\{(\text{nobody} \circ \text{'s mother}) \circledcirc ((1 \triangleright \text{Alice}) \triangleright \text{liked})\} \vdash S \\ \hline\hline \Gamma\{\text{nobody} \circledcirc (\text{'s mother} \triangleleft ((1 \triangleright \text{Alice}) \triangleright \text{liked}))\} \vdash S \end{array} \begin{array}{l} \text{Root} \\ \\ \text{Right} \\ \\ \text{Right} \\ \\ \text{Left} \end{array}$$



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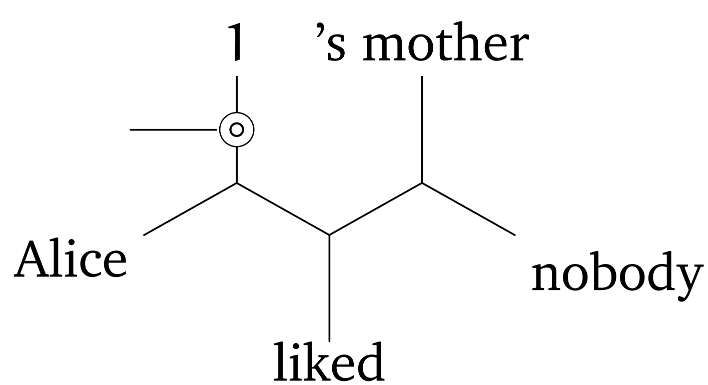
$$\begin{array}{c}\Gamma\{\text{Alice} \circ (\text{liked} \circ (\text{nobody} \circ \text{'s mother}))\} \vdash S \\ \hline\hline \Gamma\{(\text{Alice} \circ (\text{liked} \circ (\text{nobody} \circ \text{'s mother}))) \circledcirc 1\} \vdash S \\ \hline\hline \Gamma\{(\text{liked} \circ (\text{nobody} \circ \text{'s mother})) \circledcirc (1 \triangleright \text{Alice})\} \vdash S \\ \hline\hline \Gamma\{(\text{nobody} \circ \text{'s mother}) \circledcirc ((1 \triangleright \text{Alice}) \triangleright \text{liked})\} \vdash S \\ \hline\hline \Gamma\{\text{nobody} \circledcirc (\text{'s mother} \triangleleft ((1 \triangleright \text{Alice}) \triangleright \text{liked}))\} \vdash S\end{array}\begin{array}{l}\\ \text{Root}\\ \\ \text{Right}\\ \\ \text{Right}\\ \\ \text{Left}\\ \\ \end{array}$$



Delimited continuations: structural postulates

$$\frac{\Gamma\{A\} \vdash S}{\Gamma\{A \circledcirc 1\} \vdash S}\ \text{Root}$$

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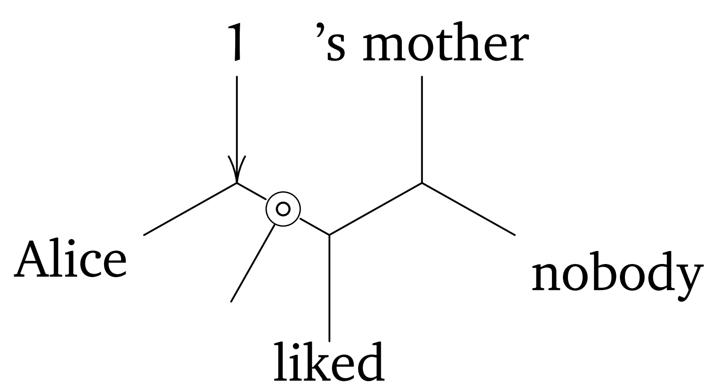
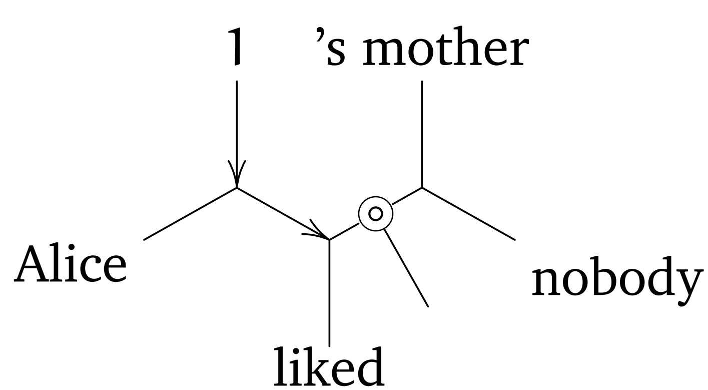


Delimited continuations: structural postulates

$$\begin{array}{c} \Gamma\{A\} \vdash S \\ \hline\hline \Gamma\{A \circledcirc 1\} \vdash S \end{array}\ \text{Root} \qquad\qquad \begin{array}{c} \Gamma\{A \circledcirc (B \triangleleft C)\} \vdash S \\ \hline\hline \Gamma\{(A \circ B) \circledcirc C\} \vdash S \\ \hline\hline \Gamma\{B \circledcirc (C \triangleright A)\} \vdash S \end{array}\ \begin{array}{l} \text{Left} \\ \text{Right} \end{array}$$

$$\begin{array}{c} \Gamma\Big\{\text{Alice} \circ (\text{liked} \circ (\text{nobody} \circ \text{'s mother}))\Big\} \vdash S \\ \hline\hline \Gamma\Big\{\big(\text{Alice} \circ (\text{liked} \circ (\text{nobody} \circ \text{'s mother}))\big) \circledcirc 1\Big\} \vdash S \\ \hline\hline \Gamma\Big\{\big(\text{liked} \circ (\text{nobody} \circ \text{'s mother})\big) \circledcirc (1 \triangleright \text{Alice})\Big\} \vdash S \\ \hline\hline \Gamma\Big\{(\text{nobody} \circ \text{'s mother}) \circledcirc \big((1 \triangleright \text{Alice}) \triangleright \text{liked}\big)\Big\} \vdash S \\ \hline\hline \Gamma\Big\{\text{nobody} \circledcirc \big(\text{'s mother} \triangleleft ((1 \triangleright \text{Alice}) \triangleright \text{liked})\big)\Big\} \vdash S \end{array}\ \begin{array}{l} \text{Root} \\ \text{Right} \\ \text{Right} \\ \text{Left} \end{array}$$

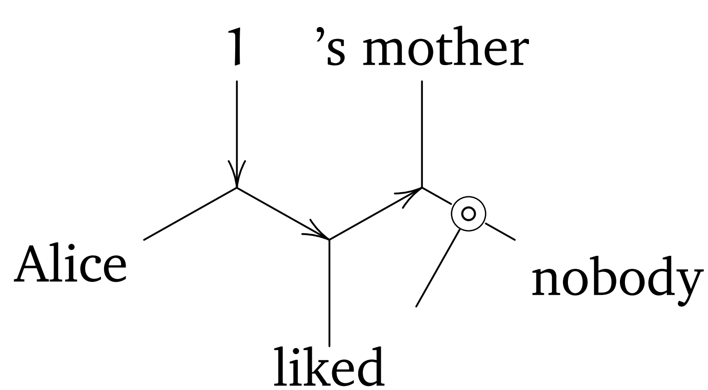


Delimited continuations: structural postulates

$$\frac{\Gamma\{A\} \vdash S}{\Gamma\{A \circledcirc 1\} \vdash S}\ \text{Root}$$

$$\frac{\frac{\Gamma\{A \circledcirc (B \triangleleft C)\} \vdash S}{\Gamma\{(A \circ B) \circledcirc C\} \vdash S}\ \text{Left}}{\Gamma\{B \circledcirc (C \triangleright A)\} \vdash S}\ \text{Right}$$

$$\Gamma\{\text{Alice} \circ (\text{liked} \circ (\text{nobody} \circ \text{'s mother}))\} \vdash S$$
$$\overline{\overline{}}\ \text{Root}$$
$$\Gamma\{(\text{Alice} \circ (\text{liked} \circ (\text{nobody} \circ \text{'s mother}))) \circledcirc 1\} \vdash S$$
$$\overline{\overline{}}\ \text{Right}$$
$$\Gamma\{(\text{liked} \circ (\text{nobody} \circ \text{'s mother})) \circledcirc (1 \triangleright \text{Alice})\} \vdash S$$
$$\overline{\overline{}}\ \text{Right}$$
$$\Gamma\{(\text{nobody} \circ \text{'s mother}) \circledcirc ((1 \triangleright \text{Alice}) \triangleright \text{liked})\} \vdash S$$
$$\overline{\overline{}}\ \text{Left}$$
$$\Gamma\{\text{nobody} \circledcirc (\text{'s mother} \triangleleft ((1 \triangleright \text{Alice}) \triangleright \text{liked}))\} \vdash S$$



Delimited continuations for in-situ quantification

$$\dfrac{\text{Alice}\vdash np \qquad \dfrac{\text{liked}\vdash (np\backslash s)/np \qquad \dfrac{\dfrac{}{np\vdash np}\,\text{Axiom} \qquad \text{'s mother}\vdash np\backslash np}{np\circ\text{'s mother}\vdash np}\,\backslash\text{E}}{\text{liked}\circ(np\circ\text{'s mother})\vdash np\backslash s}\,/\text{E}}{\text{Alice}\circ\big(\text{liked}\circ(np\circ\text{'s mother})\big)\vdash s}\,\backslash\text{E}$$

$$\vdots\ \text{Root,Right,Right,Left}$$

$$\dfrac{\boxed{\text{nobody}\vdash s\mathbin{/\!\!/}(np\mathbin{\backslash\!\!\backslash}s)} \qquad \dfrac{np\circledcirc\big(\text{'s mother}\triangleleft((1\triangleright\text{Alice})\triangleright\text{liked})\big)\vdash s}{\text{'s mother}\triangleleft\big((1\triangleright\text{Alice})\triangleright\text{liked}\big)\vdash np\mathbin{\backslash\!\!\backslash}s}\,\mathbin{\backslash\!\!\backslash}\text{I}}{\text{nobody}\circledcirc\big(\text{'s mother}\triangleleft((1\triangleright\text{Alice})\triangleright\text{liked})\big)\vdash s}\,\mathbin{/\!\!/}\text{E}$$

$$\vdots\ \text{Left,Right,Right,Root}$$

$$\text{Alice}\circ\big(\text{liked}\circ(\text{nobody}\circ\text{'s mother})\big)\vdash s$$

Ambiguity in delimitation: “Alice told Bob to criticize nobody’s mother.”

Delimited continuations for in-situ quantifications

$$\vdots$$

$$np \circ \big(\text{liked} \circ (np \circ \text{'s mother})\big) \vdash s$$

$$\vdots \; \scriptstyle \text{Root,Right,Right,Left}$$

$$\dfrac{\text{nobody} \vdash s /\!\!/ (np \backslash\!\!\backslash s) \qquad \dfrac{np \circledcirc \big(\text{'s mother} \lhd ((1 \rhd np) \rhd \text{liked})\big) \vdash s}{\text{'s mother} \lhd \big((1 \rhd np) \rhd \text{liked}\big) \vdash np \backslash\!\!\backslash s}\,\backslash\!\!\backslash \mathrm{I}}{\text{nobody} \circledcirc \big(\text{'s mother} \lhd ((1 \rhd np) \rhd \text{liked})\big) \vdash s}\,/\!\!/ \mathrm{E}$$

$$\vdots \; \scriptstyle \text{Left,Right,Right,Left}$$

$$\dfrac{\text{somebody} \vdash s /\!\!/ (np \backslash\!\!\backslash s) \qquad \dfrac{np \circledcirc \big((\text{liked} \circ (\text{nobody} \circ \text{'s mother})) \lhd 1\big) \vdash s}{(\text{liked} \circ (\text{nobody} \circ \text{'s mother})) \lhd 1 \vdash np \backslash\!\!\backslash s}\,\backslash\!\!\backslash \mathrm{I}}{\text{somebody} \circledcirc \big((\text{liked} \circ (\text{nobody} \circ \text{'s mother})) \lhd 1\big) \vdash s}\,/\!\!/ \mathrm{E}$$

$$\vdots \; \scriptstyle \text{Left,Root}$$

$$\text{somebody} \circ \big(\text{liked} \circ (\text{nobody} \circ \text{'s mother})\big) \vdash s$$

The quantifier evaluated earlier takes wider scope in the syntactic proof and the truth-conditional meaning.

Polarity sensitivity

The quantifiers “a”, “some”, and “any”† all look existential:

Did a student call?
Did some student call?
Did any student call?

$$\exists x.\, \mathsf{student}(x) \wedge \mathsf{called}(x)$$

But do not behave the same:

No student liked some course.	(unambiguous $\exists\neg$)
No student liked a course.	(ambiguous $\neg\exists$, $\exists\neg$)
No student liked any course.	(unambiguous $\neg\exists$)
Some student liked no course.	(unambiguous $\exists\neg$)
A student liked no course.	(ambiguous $\neg\exists$, $\exists\neg$)
*Any student liked no course.	(unacceptable)

“Any” is a ***negative polarity item***:
Very roughly, it requires negative contexts, such as in the scope of “no”.

“Some” is a ***positive polarity item***:
Very roughly, it is allergic to negative contexts.

Meaning affects ambiguity and acceptability! But linear order matters too.

Unary polarities

Define three types for sentences of varying polarity:

$$\begin{aligned} s^\circ &= \langle \mathrm{r} \rangle s && \text{(verbs now return this type)} \\ s^+ &= \langle \mathrm{r} \rangle [\bar{\mathrm{p}}] \langle \mathrm{p} \rangle s \\ s^- &= [\bar{\mathrm{p}}] \langle \mathrm{p} \rangle \langle \mathrm{r} \rangle s \end{aligned}$$

Unary type constructors like $\langle \mathrm{p} \rangle$ and $[\bar{\mathrm{p}}]$ come in pairs. Each pair is an adjunction.

Additional natural-deduction rules for unary connectives (focus on polarity)

$$\frac{\Gamma \vdash A}{\langle \mathrm{p} \rangle \Gamma \vdash \langle \mathrm{p} \rangle A}\ \langle \mathrm{p} \rangle \mathrm{I} \qquad \frac{\Delta \vdash \langle \mathrm{p} \rangle A \quad \Gamma\{\langle \mathrm{p} \rangle A\} \vdash B}{\Gamma\{\Delta\} \vdash B}\ \langle \mathrm{p} \rangle \mathrm{E}$$

$$\frac{\langle \mathrm{p} \rangle \Gamma \vdash A}{\Gamma \vdash [\bar{\mathrm{p}}] A}\ [\bar{\mathrm{p}}] \mathrm{I} \qquad \frac{\Gamma \vdash [\bar{\mathrm{p}}] A}{\langle \mathrm{p} \rangle \Gamma \vdash A}\ [\bar{\mathrm{p}}] \mathrm{E}$$

A handy lemma:

$$\dfrac{\dfrac{}{\langle \mathrm{p} \rangle A \vdash \langle \mathrm{p} \rangle A}\ \text{Axiom}}{A \vdash [\bar{\mathrm{p}}] \langle \mathrm{p} \rangle A}\ [\bar{\mathrm{p}}] \mathrm{I}$$

Hence $s^\circ \vdash s^+$ and $s^\circ \vdash s^-$.

Possibility

$\langle \mathrm{p} \rangle A$ *means* "A for some p-child"
$\langle \mathrm{q} \rangle A$ *means* "A for some q-child"
$\langle \mathrm{r} \rangle A$ *means* "A for some r-child"

Necessity

$[\bar{\mathrm{p}}] A$ *means* "A for every p-parent"
$[\bar{\mathrm{q}}] A$ *means* "A for every q-parent"
$[\bar{\mathrm{r}}] A$ *means* "A for every r-parent"

Unary modalities for polarity sensitivity

$$\vdots$$

$$\dfrac{\dfrac{np \circ (\text{liked} \circ (np \circ \text{'s mother})) \vdash s^{\circ}}{\langle \mathrm{p} \rangle (np \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash \langle \mathrm{p} \rangle s^{\circ}}\ \langle \mathrm{p} \rangle \mathrm{I}}{np \circ (\text{liked} \circ (np \circ \text{'s mother})) \vdash s^{-}}\ [\bar{\mathrm{p}}]\mathrm{I}$$

$$\vdots\ \text{Root,Right,Right,Left}$$

$$\dfrac{\text{anybody} \vdash s^{-} /\!\!/ (np \backslash\!\!\backslash s^{-}) \qquad \dfrac{np \circledcirc (\text{'s mother} \triangleleft ((1 \triangleright np) \triangleright \text{liked})) \vdash s^{-}}{\text{'s mother} \triangleleft ((1 \triangleright np) \triangleright \text{liked}) \vdash np \backslash\!\!\backslash s^{-}}\ \backslash\!\!\backslash \mathrm{I}}{\text{anybody} \circledcirc (\text{'s mother} \triangleleft ((1 \triangleright np) \triangleright \text{liked})) \vdash s^{-}}\ /\!\!/ \mathrm{E}$$

$$\vdots\ \text{Left,Right,Right,Left}$$

$$\dfrac{\text{nobody} \vdash s^{\circ} /\!\!/ (np \backslash\!\!\backslash s^{-}) \qquad \dfrac{np \circledcirc ((\text{liked} \circ (\text{anybody} \circ \text{'s mother})) \triangleleft 1) \vdash s^{-}}{(\text{liked} \circ (\text{anybody} \circ \text{'s mother})) \triangleleft 1 \vdash np \backslash\!\!\backslash s^{-}}\ \backslash\!\!\backslash \mathrm{I}}{\text{nobody} \circledcirc ((\text{liked} \circ (\text{anybody} \circ \text{'s mother})) \triangleleft 1) \vdash s^{\circ}}\ /\!\!/ \mathrm{E}$$

$$\vdots\ \text{Left,Root}$$

$$\text{nobody} \circ (\text{liked} \circ (\text{anybody} \circ \text{'s mother})) \vdash s^{\circ}$$

Unary modalities for polarity sensitivity, cont'd

A sequent is complete iff its antecedent is built up using the $\circ$ connective only and its conclusion is of the form $\langle \mathrm{r} \rangle A$ (that is, either s° or s^+).

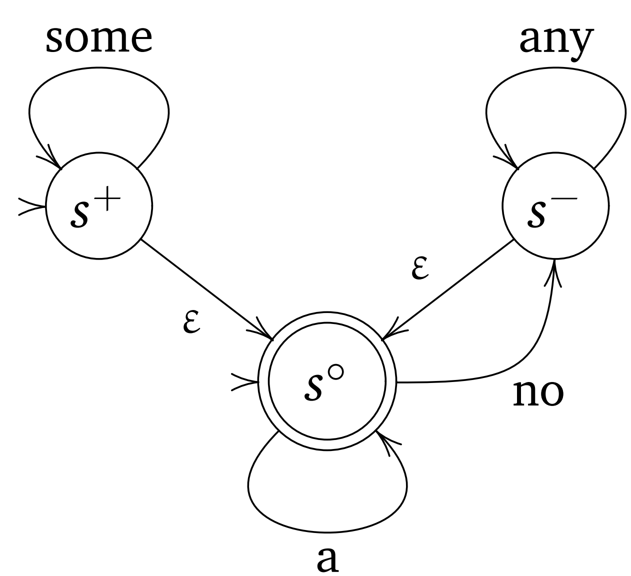
$$
\dfrac{\text{anybody} \vdash s^- /\!\!/ (np \backslash\!\!\backslash s^-) \qquad \dfrac{\dfrac{\begin{array}{c}\dfrac{\dfrac{\begin{array}{c}\vdots \\ \text{Alice} \circ (\text{liked} \circ (np \circ \text{'s mother})) \vdash s^\circ\end{array}}{\langle \mathrm{p} \rangle (\text{Alice} \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash \langle \mathrm{p} \rangle s^\circ}\,\langle \mathrm{p} \rangle\mathrm{I}}{\text{Alice} \circ (\text{liked} \circ (np \circ \text{'s mother})) \vdash s^-}\,[\bar{\mathrm{p}}]\mathrm{I} \\ \vdots\ \scriptstyle\text{Root,Right,Right,Left}\end{array}}{np \circledcirc (\text{'s mother} \triangleleft ((1 \triangleright \text{Alice}) \triangleright \text{liked})) \vdash s^-}}{\text{'s mother} \triangleleft ((1 \triangleright \text{Alice}) \triangleright \text{liked}) \vdash np \backslash\!\!\backslash s^-}\,\backslash\!\!\backslash\mathrm{I}}{\text{anybody} \circledcirc (\text{'s mother} \triangleleft ((1 \triangleright \text{Alice}) \triangleright \text{liked})) \vdash s^-}\,/\!\!/\mathrm{E}
$$

*Alice liked anybody's mother.
Nobody said that Alice liked anybody's mother.

Unary modalities for polarity sensitivities

$$s^\circ \vdash s^+$$
$$s^\circ \vdash s^-$$

$$\text{nobody} \vdash s^\circ /\!\!/ (np \backslash\!\!\backslash s^-)$$
$$\text{anybody} \vdash s^- /\!\!/ (np \backslash\!\!\backslash s^-)$$
$$\text{somebody} \vdash s^+ /\!\!/ (np \backslash\!\!\backslash s^+)$$
$$\text{a woman} \vdash s^\circ /\!\!/ (np \backslash\!\!\backslash s^\circ)$$



No student liked some course.	(unambiguous $\exists\neg$)	✓
No student liked a course.	(ambiguous $\neg\exists$, $\exists\neg$)	✓
No student liked any course.	(unambiguous $\neg\exists$)	✓
Some student liked no course.	(unambiguous $\exists\neg$)	✓
A student liked no course.	(ambiguous $\neg\exists$, $\exists\neg$)	✓
*Any student liked no course.	(unacceptable)	×

Prediction: the quantifiers in a sentence must form a valid transition sequence, from widest to narrowest scope.

Evaluation order

These structural postulates in multimodal categorial grammar:

$$\frac{\Gamma\{A\} \vdash S}{\Gamma\{A \circledcirc 1\} \vdash S}\,\text{Root} \qquad \frac{\Gamma\{(A \circ B) \circledcirc C\} \vdash S}{\Gamma\{A \circledcirc (B \triangleleft C)\} \vdash S}\,\text{Left} \qquad \frac{\Gamma\{(A \circ B) \circledcirc C\} \vdash S}{\Gamma\{B \circledcirc (C \triangleright A)\} \vdash S}\,\text{Right}$$

encode this recursive definition of evaluation contexts:

$$C ::= \{\ \} \quad | \quad C\{\{\ \} \circ B\} \quad | \quad C\{A \circ \{\ \}\}.$$

Modify the Right rule:

$$\frac{\Gamma\{A\} \vdash S}{\Gamma\{A \circledcirc 1\} \vdash S}\,\text{Root} \qquad \frac{\Gamma\{(A \circ B) \circledcirc C\} \vdash S}{\Gamma\{A \circledcirc (B \triangleleft C)\} \vdash S}\,\text{Left} \qquad \frac{\Gamma\{(\langle q \rangle A \circ B) \circledcirc C\} \vdash S}{\Gamma\{B \circledcirc (C \triangleright \langle q \rangle A)\} \vdash S}\,\text{Right}$$

to enforce ***left-to-right evaluation*** in evaluation contexts:

$$C ::= \{\ \} \quad | \quad C\{\{\ \} \circ B\} \quad | \quad C\{\langle q \rangle A \circ \{\ \}\}.$$

Here we mark pure values with the unary prefix $\langle q \rangle$.

Staging

The $\langle \mathrm{q} \rangle$ modality stands for ***quotation*** of programs. Any program can be quoted:

$$\frac{\Gamma\{\langle \mathrm{q} \rangle A\} \vdash S}{\Gamma\{A\} \vdash S}\,\text{Quote (T)}$$

And any two programs can be concatenated:

$$\frac{\Gamma\{\langle \mathrm{q} \rangle (A \circ \mathrm{B})\} \vdash S}{\Gamma\{\langle \mathrm{q} \rangle A \circ \langle \mathrm{q} \rangle \mathrm{B}\} \vdash S}\,\text{Concat (K}'\text{)}$$

But only complete programs (with types of the form $\langle \mathrm{r} \rangle A$) can be unquoted (run):

$$\frac{\Gamma\{\langle \mathrm{r} \rangle A\} \vdash S}{\Gamma\{\langle \mathrm{q} \rangle \langle \mathrm{r} \rangle A\} \vdash S}\,\text{Run}$$

These stipulations interact to make correct linguistic predictions.

Nobody liked a woman's mother	(ambiguous $\neg\exists$, $\exists\neg$)
*Anybody liked nobody's mother	(unacceptable)

Evaluation order and staging for linear scope

First of all, “nobody likes a woman’s mother” can take linear scope. This is easy under left-to-right evaluation, because quantifiers evaluated earlier scope wider.

$$
\dfrac{\dfrac{\dfrac{\vdots}{np \circ (\text{liked} \circ (np \circ \text{'s mother})) \vdash s^{\circ}}}{\langle \mathrm{q} \rangle (np \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash \langle \mathrm{q} \rangle s^{\circ}}\,\langle \mathrm{q} \rangle \mathrm{I} \qquad \dfrac{\dfrac{}{s^{\circ} \vdash s^{\circ}}\,\text{Axiom}}{\langle \mathrm{q} \rangle s^{\circ} \vdash s^{\circ}}\,\text{Run}}{\langle \mathrm{q} \rangle (np \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash s^{\circ}}\,\langle \mathrm{q} \rangle \mathrm{E}
$$

$$
\dfrac{\dfrac{\langle \mathrm{q} \rangle (np \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash s^{\circ}}{\langle \mathrm{q} \rangle np \circ (\langle \mathrm{q} \rangle \text{liked} \circ \langle \mathrm{q} \rangle (np \circ \text{'s mother})) \vdash s^{\circ}}\,\text{Concat} \times 2}{\langle \mathrm{q} \rangle np \circ (\langle \mathrm{q} \rangle \text{liked} \circ (np \circ \text{'s mother})) \vdash s^{\circ}}\,\text{Quote}
$$

$$
\vdots \ \text{Root,Right,Right,Left,}\backslash\!\!\backslash\text{I,}/\!\!/\text{E,Left,Right,Right,Root}
$$

$$
\dfrac{\dfrac{\dfrac{\langle \mathrm{q} \rangle np \circ (\langle \mathrm{q} \rangle \text{liked} \circ (\text{a woman} \circ \text{'s mother})) \vdash s^{\circ}}{\langle \mathrm{p} \rangle (\langle \mathrm{q} \rangle np \circ (\langle \mathrm{q} \rangle \text{liked} \circ (\text{a woman} \circ \text{'s mother}))) \vdash \langle \mathrm{p} \rangle s^{\circ}}\,\langle \mathrm{p} \rangle \mathrm{I}}{\langle \mathrm{q} \rangle np \circ (\langle \mathrm{q} \rangle \text{liked} \circ (\text{a woman} \circ \text{'s mother})) \vdash s^{-}}\,[\bar{\mathrm{p}}]\mathrm{I}}{np \circ (\text{liked} \circ (\text{a woman} \circ \text{'s mother})) \vdash s^{-}}\,\text{Quote} \times 2
$$

$$
\vdots \ \text{Root,Left,}\backslash\!\!\backslash\text{I,}/\!\!/\text{E,Left,Root}
$$

$$
\text{nobody} \circ (\text{liked} \circ (\text{a woman} \circ \text{'s mother})) \vdash s^{-}
$$

Evaluation order and staging for inverse scope

Moreover, “nobody likes a woman’s mother” can also take inverse scope, because the answer type $s^\circ = \langle \mathrm{r}\rangle s$ returned by “nobody” can be unquoted.

$$\vdots$$

$$\dfrac{\dfrac{np \circ (\text{liked} \circ (np \circ \text{'s mother})) \vdash s^\circ}{\langle \mathrm{p}\rangle(np \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash \langle \mathrm{p}\rangle s^\circ}\,\langle \mathrm{p}\rangle\mathrm{I}}{np \circ (\text{liked} \circ (np \circ \text{'s mother})) \vdash s^-}\,[\bar{\mathrm{p}}]\mathrm{I}$$

$$\vdots\ \text{Root,Left,}\mathbin{\backslash\!\backslash}\text{I,}\mathbin{/\!/}\text{E,Left,Root}$$

$$\dfrac{\dfrac{\text{nobody} \circ (\text{liked} \circ (np \circ \text{'s mother})) \vdash s^\circ}{\langle \mathrm{q}\rangle(\text{nobody} \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash \langle \mathrm{q}\rangle s^\circ}\,\langle \mathrm{q}\rangle\mathrm{I} \qquad \dfrac{\dfrac{}{s^\circ \vdash s^\circ}\,\text{Axiom}}{\langle \mathrm{q}\rangle s^\circ \vdash s^\circ}\,\text{Run}}{\langle \mathrm{q}\rangle(\text{nobody} \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash s^\circ}\,\langle \mathrm{q}\rangle\mathrm{E}$$

$$\dfrac{\dfrac{\langle \mathrm{q}\rangle(\text{nobody} \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash s^\circ}{\langle \mathrm{q}\rangle\text{nobody} \circ (\langle \mathrm{q}\rangle\text{liked} \circ \langle \mathrm{q}\rangle(np \circ \text{'s mother})) \vdash s^\circ}\,\text{Concat} \times 2}{\langle \mathrm{q}\rangle\text{nobody} \circ (\langle \mathrm{q}\rangle\text{liked} \circ (np \circ \text{'s mother})) \vdash s^\circ}\,\text{Quote}$$

$$\vdots\ \text{Root,Right,Right,Left,}\mathbin{\backslash\!\backslash}\text{I,}\mathbin{/\!/}\text{E,Left,Right,Right,Root}$$

$$\dfrac{\langle \mathrm{q}\rangle\text{nobody} \circ (\langle \mathrm{q}\rangle\text{liked} \circ (\text{a woman} \circ \text{'s mother})) \vdash s^\circ}{\text{nobody} \circ (\text{liked} \circ (\text{a woman} \circ \text{'s mother})) \vdash s^\circ}\,\text{Quote} \times 2$$

Evaluation order and staging for polarity sensitivity

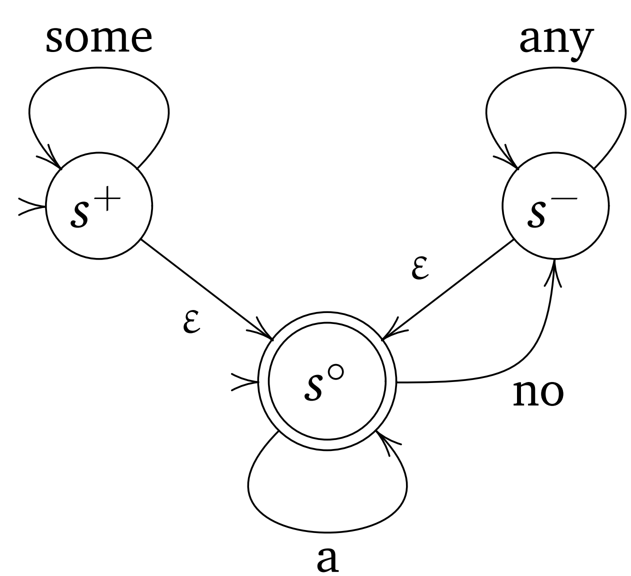
Nevertheless, “anybody likes nobody’s mother” cannot take inverse scope, because the answer type $s^- = [\bar{\mathrm{p}}]\langle\mathrm{p}\rangle\langle\mathrm{r}\rangle s$ returned by “anybody” cannot be unquoted.

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{\vdots}{np \circ (\text{liked} \circ (np \circ \text{'s mother})) \vdash s^\circ}
}{\langle\mathrm{p}\rangle(np \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash \langle\mathrm{p}\rangle s^\circ}\langle\mathrm{p}\rangle\mathrm{I}
}{np \circ (\text{liked} \circ (np \circ \text{'s mother})) \vdash s^-}[\bar{\mathrm{p}}]\mathrm{I}
\ \vdots\ \text{Root,Left,}\backslash\!\!\backslash\text{I,}/\!\!/\text{E,Left,Root}
}{\text{anybody} \circ (\text{liked} \circ (np \circ \text{'s mother})) \vdash s^-}
}{\langle\mathrm{q}\rangle(\text{anybody} \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash \langle\mathrm{q}\rangle s^-}\langle\mathrm{q}\rangle\mathrm{I}
\quad
\cancel{\boxed{\dfrac{\dfrac{}{s^- \vdash s^-}\text{Axiom}}{\langle\mathrm{q}\rangle s^- \vdash s^-}\text{Run}}}
}{
\dfrac{
\dfrac{
\dfrac{\langle\mathrm{q}\rangle(\text{anybody} \circ (\text{liked} \circ (np \circ \text{'s mother}))) \vdash s^-}{\langle\mathrm{q}\rangle\text{anybody} \circ (\langle\mathrm{q}\rangle\text{liked} \circ \langle\mathrm{q}\rangle(np \circ \text{'s mother})) \vdash s^-}\text{Concat} \times 2
}{\langle\mathrm{q}\rangle\text{anybody} \circ (\langle\mathrm{q}\rangle\text{liked} \circ (np \circ \text{'s mother})) \vdash s^-}\text{Quote}
\ \vdots\ \text{Root,Right,Right,Left,}\backslash\!\!\backslash\text{I,}/\!\!/\text{E,Left,Right,Right,Root}
}{
\dfrac{\langle\mathrm{q}\rangle\text{anybody} \circ (\langle\mathrm{q}\rangle\text{liked} \circ (\text{nobody} \circ \text{'s mother})) \vdash s^\circ}{\text{anybody} \circ (\text{liked} \circ (\text{nobody} \circ \text{'s mother})) \vdash s^\circ}\text{Quote} \times 2
}
}\langle\mathrm{q}\rangle\mathrm{E}
$$

An old puzzle solved

$$s^\circ \vdash s^+$$
$$s^\circ \vdash s^-$$

$$\text{nobody} \vdash s^\circ /\!\!/ (np \backslash\!\!\backslash s^-)$$
$$\text{anybody} \vdash s^- /\!\!/ (np \backslash\!\!\backslash s^-)$$
$$\text{somebody} \vdash s^+ /\!\!/ (np \backslash\!\!\backslash s^+)$$
$$\text{a woman} \vdash s^\circ /\!\!/ (np \backslash\!\!\backslash s^\circ)$$



No student liked some course.	(unambiguous ∃¬)	✓
No student liked a course.	(ambiguous ¬∃, ∃¬)	✓
No student liked any course.	(unambiguous ¬∃)	✓
Some student liked no course.	(unambiguous ∃¬)	✓
A student liked no course.	(ambiguous ¬∃, ∃¬)	✓
*Any student liked no course.	(unacceptable)	✓

Prediction: the quantifiers in a sentence must form a valid transition sequence, from widest to narrowest scope.

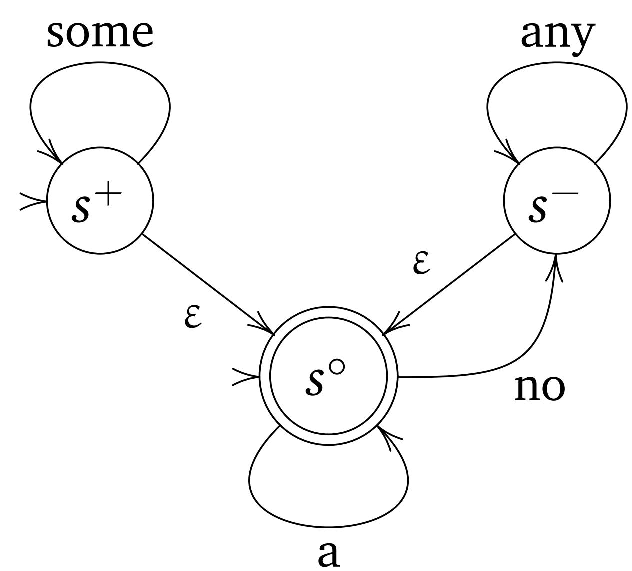
Also, inverse-scope transitions must pass through a start state.

An old puzzle solved, and a new one

$$s^\circ \vdash s^+$$

$$s^\circ \vdash s^-$$

$$\begin{aligned}
\text{nobody} &\vdash s^\circ /\!\!/ (np \backslash\!\!\backslash s^-) \\
\text{anybody} &\vdash s^- /\!\!/ (np \backslash\!\!\backslash s^-) \\
\text{somebody} &\vdash s^+ /\!\!/ (np \backslash\!\!\backslash s^+) \\
\text{a woman} &\vdash s^\circ /\!\!/ (np \backslash\!\!\backslash s^\circ) \\
\text{everybody} &\vdash ???
\end{aligned}$$



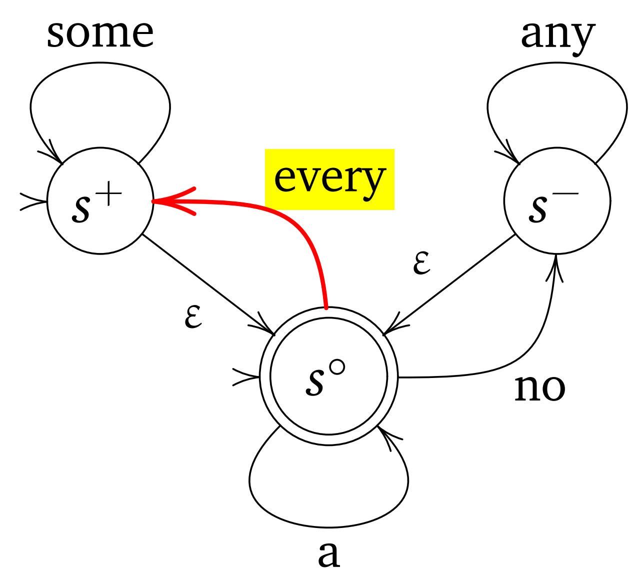
What is the type of "everybody"? Hint:

"A woman introduced everybody to somebody": linear scope ok.

An old puzzle solved, and a new one

$$s^\circ \vdash s^+$$
$$s^\circ \vdash s^-$$

$$\text{nobody} \vdash s^\circ \mathbin{/\!\!/} (np \mathbin{\backslash\!\!\backslash} s^-)$$
$$\text{anybody} \vdash s^- \mathbin{/\!\!/} (np \mathbin{\backslash\!\!\backslash} s^-)$$
$$\text{somebody} \vdash s^+ \mathbin{/\!\!/} (np \mathbin{\backslash\!\!\backslash} s^+)$$
$$\text{a woman} \vdash s^\circ \mathbin{/\!\!/} (np \mathbin{\backslash\!\!\backslash} s^\circ)$$
$$\text{everybody} \vdash s^\circ \mathbin{/\!\!/} (np \mathbin{\backslash\!\!\backslash} s^+)$$



What is the type of “everybody”? Hint:

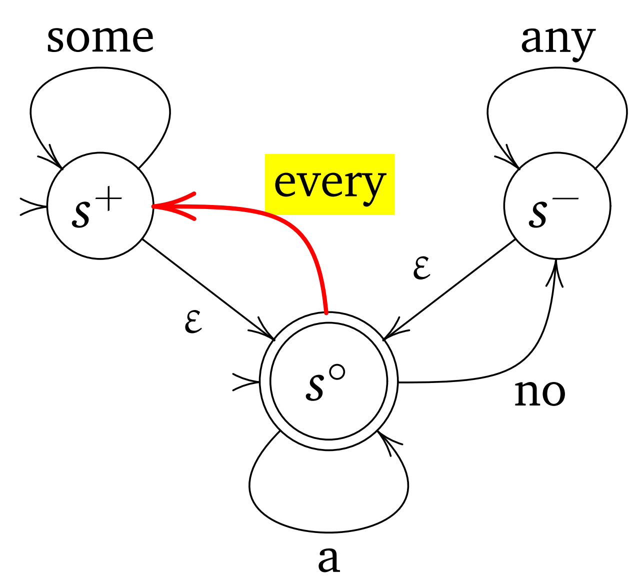
“A woman introduced everybody to somebody”: linear scope ok.

So “every” must turn the output of “some” into the input of “a”.

An old puzzle solved, and a new one

$$s^\circ \vdash s^+$$
$$s^\circ \vdash s^-$$

$$\begin{aligned}
\text{nobody} &\vdash s^\circ /\!\!/ (np \backslash\!\!\backslash s^-) \\
\text{anybody} &\vdash s^- /\!\!/ (np \backslash\!\!\backslash s^-) \\
\text{somebody} &\vdash s^+ /\!\!/ (np \backslash\!\!\backslash s^+) \\
\text{a woman} &\vdash s^\circ /\!\!/ (np \backslash\!\!\backslash s^\circ) \\
\text{everybody} &\vdash s^\circ /\!\!/ (np \backslash\!\!\backslash s^+)
\end{aligned}$$



What is the type of “everybody”? Hint:

“A woman introduced everybody to somebody”: linear scope ok.

So “every” must turn the output of “some” into the input of “a”.

Confirmation:

“Nobody introduced Alice to somebody”: linear scope bad.
“Nobody introduced everybody to somebody”: linear scope ok again!

Outline

OLD	Multiplicative linear logic *for* linguistics	Alice liked Bob.
NEW	Delimited continuations *for* quantification	Alice liked everybody's mother.
OLD	Unary modalities *for* polarity sensitivity	*Alice liked anybody's mother. Nobody liked anybody's mother.
NEW	Evaluation order *for* linear precedence	*Anybody liked nobody's mother.
NEW	Staging *for* scope ambiguities	Somebody liked everybody's mother.

Payoffs

For linguistics:

- Cover more empirical data.
- Relate (denotational) semantics to (operational) psycholinguistics?

For computer science:

- Understand delimited continuations geometrically and logically.
- Staging side effects?