MEANINGS OF MULTIPLE-WH QUESTIONS

CHUNG-CHIEH SHAN

Abstract. Questions containing multiple wh-phrases, in particular singular wh-phrases, are problematic in the semantics of interrogatives. The central puzzle, known as the issue of *pair-list questions*, is why *which man saw Mary* presumes that only one man saw Mary, yet *which man saw which woman* does not presume that only one man saw a woman. Discourse and pragmatic factors complicate the picture, as does wide variation among ideolects. We present our empirical observations and argue that they cannot be accounted for by Higginbotham and May's previous account, based on a proposed absorption operation on adjacent wh-variables. We then propose an alternative theory, in which we hypothesize that singular wh-phrases are ambiguous between a meaning presupposing singularity and a meaning similar to that of *each*.

1. Introduction

A semantics for interrogatives needs to take into account presupposition on the part of the speaker and answers on the part of the listener. It also needs to accord with observed syntactic phenomena. A simple example is (1), which presupposes that exactly one man saw Mary among the men under discussion.

(1) Which man saw Mary?

The meaning of (1) is different from that of (2), even though at first glance they ask for the same information.

(2) Which men saw Mary?

We can distinguish between (1) and (2) empirically in a variety of ways. A simple approach is to assume that, if the speaker of a question has no definite knowledge that an assertion is true, then the question should not presuppose the assertion:

(3) a. I'm sure some men on the street saw Mary. *Which man saw Mary?
b. I'm sure some men on the street saw Mary. Which men saw Mary?

Another way is to rely on the speaker to never presuppose the falsehood of a preceding statement:

(4) a. Come on. There were over fifty men in attendance, and at least half of the men in the room saw Mary. *Which man saw Mary?
b. Come on. There were over fifty men in attendance, and at least half of the men in the room saw Mary. Which men saw Mary?

A third way is to rely on the listener to never respond to a question with a reply that does not inform beyond what is already presupposed by the question:

Date: January 25, 2001.
This paper is jointly submitted as a term paper for Linguistics 118 (Susumu Kuno, Introduction to discourse analysis) and for Linguistics 205 (Jonathan Nissenbaum, Topics at the syntax-semantics interface) in Fall 2000 at Harvard University.

(5) a. Which man saw Mary?—*I'm not sure, but I do know that it's exactly one man.
b. Which men saw Mary?—I'm not sure, but I do know that it's exactly one man.

Finally, note that *well, actually* in front of a reply is questionable unless the reply contradicts the presupposition of the question:

(6) a. Which man saw Mary?—Well, actually, both Bill and John saw Mary.
b. Which men saw Mary?—?Well, actually, both Bill and John saw Mary.
c. Which men saw Mary?—*Well, actually, there were quite a few men who saw Mary, but I have no idea who exactly.

2. Direct wh-questions

In this paper, I focus on direct questions where one or more wh-phrases (such as *which man*) occur in a simple predicate (such as *saw*). Most of the discussion probably applies to indirect questions (i.e., embedded interrogatives) as well. Direct questions can be *single-wh questions*, as in (7a), or *multiple-wh questions*, as in (7b).

(7) a. (i) Which man saw Mary?
(ii) Which tools did they threaten Kenny with?
b. (i) Which man saw which woman?
(ii) Which bag contains which marbles?
(iii) Which teacher gave which book to which student?

I assume that these questions correspond to LF representations in which the wh-variables are raised over the main clause. For example, I assume that, at LF, the sentences in (7) roughly correspond to (8). Note that—for the moment—I do not claim any particular ordering or commanding relationship among the wh-variables in (8b). This is indicated by the braces.[1]

(8) a. (i) [which man x] x saw Mary
(ii) [which tools x] they threatened Kenny with x
b. (i) {[which man x], [which woman y]} x saw y
(ii) {[which bag x], [which marbles y]} x contains y
(iii) {[which teacher x], [which book y], [which student z]} x gave y to z

The clausal structures in (8) over which wh-variables are raised (for example *x saw y*) are known as the *neutral core* of the question, and admits a usual truth-conditional semantic analysis [1].

In this paper we will often be concerned with differences between *singular-which* variables and *plural-which* variables, by which I mean wh-variables of the forms

(9) a. [which (plural N') x],
b. [which (singular N') x].

We denote plural wh-variables of the form (9a), for example *which women x*, by $[\mathbb{P}x]$; and singular wh-variables of the form (9b), for example *which woman x*, by $[\mathbb{S}x]$.

[1] Neither do I claim the absence of ordering or commanding relationships among the wh-variables in (8b). In fact, I will claim the presence of ordering, i.e., remove the braces in (8b), in §5.3 below.

Questions with plural wh-variables often cause complications that are specific not to wh-questions but to plural NPs. For example, (7aii) is ambiguous—it could be asking for a set of tools such that Kenny was threatened with these tools all at once, or it could be asking for a set of tools such that each tool in the set has been used in the past, possibly at different times, to threaten Kenny. For another example, consider the declarative sentence

(10) The professors met the students.

This sentence could describe a discussion panel between a group of professors and a group of students, or it could describe one-on-one advising meetings. In this paper, I do not consider these 'group readings.' I have tried to exclude them from empirical data by using predicates that have no natural group reading distinct from the individual reading (for example *contain*).

We introduce notation as follows: Let q be a direct wh-question involving the (finite) set of wh-variables V. Denote the number of variables in V by n. Let ψ be the neutral core predicate in q; we restrict our attention to when each variable in V occurs exactly once in ψ. For each variable x in V, let $\mathrm{Dom}(x)$ be the *domain* of x, i.e., the set of objects, or potential values for the variable x, under discussion. We restrict our attention to when the domain of every variable has finite and multiple cardinality, and is known to the speaker—in other words, in possible world semantics, $\mathrm{Dom}(x)$ is the same finite set in every world, and it is neither empty nor a singleton.

For any subset W of V, let $\mathrm{Dom}(W)$ be the Cartesian product of the domains of all variables in W, that is,

(11) $$\mathrm{Dom}(W) = \prod_{x \in W} \mathrm{Dom}(x).$$

In particular, then, $\mathrm{Dom}(V)$ is the Cartesian product of the domains of all variables involved in the question q. Define a *relation* on W as a subset of $\mathrm{Dom}(W)$. In particular, a relation on V—a *theory*—assigns a truth value to every element of $\mathrm{Dom}(V)$. In possible world semantics, we can think of ψ as a theory in each possible world. The speaker's presupposition is then a set of theories—the speaker presupposes that, in reality, ψ is a member of this set.

Let us illustrate these definitions with an example. In (8bi), the set of wh-variables is

(12) $$V = \{\text{which man } x, \text{which woman } y\},$$

or $V = \{x, y\}$ for short. (Note that the symbols x and y here denote the wh-variables themselves, not any particular value assigned to the variables.) In possible world semantics, the predicate $\psi(x, y)$ is true in a world if and only if x saw y in that world. If the question occurs while discussing a dinner party that occurred the previous evening, then $\mathrm{Dom}(x)$ might be the set of men attending, and $\mathrm{Dom}(y)$ the set of women attending. We can then think of ψ as the theory

(13) $$\psi = \{\, (a, b) \in \mathrm{Dom}(x, y) \mid a \text{ saw } b \,\}$$

in each possible world.

At this point a special case of the definitions above is worth mentioning. Note that the empty set of variables $\emptyset$ is trivially a subset of V. Following the definition of $\mathrm{Dom}(W)$ above, we see that the domain of $\emptyset$ is

(14) $\mathrm{Dom}(\emptyset) = \{()\}$,

that is, the singleton set containing the empty tuple. Since $\mathrm{Dom}(\emptyset)$ is a singleton set, there are only two relations on $\emptyset$, namely $\{()\}$ and $\{\}$. For reasons that will become apparent, we let $\top = \{()\}$ and $\bot = \{\}$. Intuitively, $\top$ indicates a true statement, and $\bot$ a false statement.

3. A proposed semantics

Higginbotham and May [2], building on [4], introduced a semantics for interrogatives and explained how it models presuppositions and partial answers, such as those at play in (5b). In this section, I will summarize their proposal, or rather, a simplified equivalent of their proposal specialized to the special case of direct wh-questions.

The meaning of a direct wh-question is a set of *alternatives*, each of which is a theory.[2] A question presupposes the disjunction of its alternatives; in other words, the speaker of a question presupposes that the true state of the world is a member of the set that is the meaning of the question.

3.1. **Plural wh-variables.** Under this framework, it is easy to characterize the meaning of questions involving a single plural *which*-phrase, i.e., questions of the form

(15) $[\mathbb{P}x]\,\psi(x)$.

Such a question makes no presuppositions regarding the set of objects satisfying ψ.[3] Correspondingly, our semantics should assign to (15) the meaning

(16) $2^{\mathrm{Dom}(x)} = \{\, X \subseteq \mathrm{Dom}(x) \,\}$,

the set of all relations on x. The number of alternatives here is $2^{\#\,\mathrm{Dom}(x)}$.

To assign to (15) the meaning (16), we use the following compositional method. Begin by assigning to the neutral core $\psi(x)$ the meaning $\top$. Then, assign to the raised wh-variable $[\mathbb{P}x]$ the meaning

(17) $\lambda R.\{\, B \subseteq S \times A \mid S \subseteq \mathrm{Dom}(x),\ A \in R \,\}$,

where $\times$ denotes the Cartesian product between each subset S of $\mathrm{Dom}(x)$ and each alternative A in R. Applying (17) to $\top$ gives (16), the meaning we want for (15).

The reason for this roundabout construction is clear when we consider questions with two or more plural which-phrases. A double-plural-which question has the general form

(18) $\{[\mathbb{P}x], [\mathbb{P}y]\}\,\psi(x, y)$.

Again assign to $\psi(x, y)$ the meaning $\top$, and the two raised wh-variables x and y the meanings

(19) $\lambda R.\{\, B \subseteq S \times A \mid S \subseteq \mathrm{Dom}(x),\ A \in R \,\}$

[2]In the analysis in [2], each alternative is not a theory but a set of theories that the speaker considers equivalent and has no interest in discerning between. Since this paper only considers alternatives that are singletons, I omit this additional level of complexity from discussion.

[3]It can be argued that the question presupposes the existence of at least one object satisfying ψ; however, the analysis of plural which-phrases in [2] does not generate this presupposition.

and

(20) $\lambda R.\{\, B \subseteq S \times A \mid S \subseteq \mathrm{Dom}(y),\, A \in R \,\}$,

respectively. Applying (20) to $\top$, we get $2^{\mathrm{Dom}(y)}$, the set of all relations on y, as before. Applying (19) to $2^{\mathrm{Dom}(y)}$, we get

(21) $2^{\mathrm{Dom}(x,y)}$,

the set of all relations on x and y, as desired. Note that the order of function application does not affect the meaning—we could have applied (20) to $\top$, then applied (21), and would have computed the same meaning.

3.2. **Singular wh-variables.** Let us turn to singular-which questions, i.e., those of the form

(22) $[\mathbb{S}x]\, \psi(x)$.

We would like questions of this form to presuppose that exactly one object in $\mathrm{Dom}(x)$ satisfies ψ. In other words, we would like the meaning of such a question to contain only singleton subsets of $\mathrm{Dom}(x)$, i.e.,

(23) $\{\, \{a\} \mid a \in \mathrm{Dom}(x) \,\}$.

To achieve this, we assign to the raised wh-variable $[\mathbb{S}x]$ the meaning

(24) $\lambda R.\{\, S \times A \mid S \subseteq \mathrm{Dom}(x),\, \#S = 1,\, A \in R \,\}$.

This meaning differs from the plural-which meaning (17) in that we require S to be a singleton set, and allow not arbitrary subsets of $S \times A$ but only $S \times A$ itself. Applying (24) to $\top$ gives (23), the meaning we want for (22).

Using the semantics defined in (17) and (24) to multiple-which questions containing at least one singular-which phrase gives results that are generally expected and desired. More specifically, consider questions of the forms

(25) a. $\{[\mathbb{S}x], [\mathbb{S}y]\}\, \psi(x, y)$,
b. $\{[\mathbb{S}x], [\mathbb{P}y]\}\, \psi(x, y)$,
c. $\{[\mathbb{P}x], [\mathbb{S}y]\}\, \psi(x, y)$.

The semantics described so far assigns to these questions the respective meanings

(26) a. $\{\, \{(a, b)\} \mid (a, b) \in \mathrm{Dom}(x, y) \,\}$,
b. $\{\, \{a\} \times B \mid a \in \mathrm{Dom}(x),\, B \subseteq \mathrm{Dom}(x) \,\}$,
c. $\{\, A \times \{b\} \mid A \subseteq \mathrm{Dom}(x),\, b \in \mathrm{Dom}(x) \,\}$.

Again, the order of function application does not affect the computed meaning.

In words, the semantics described so far generates the following presuppositions:

- The singular-singular question (25a) presupposes that there is a unique pair (a, b) that satisfies ψ. For example, the question *which man saw which woman* presupposes that one man saw one woman, and no other man saw any woman, and no other woman was seen by any man.
- The singular-plural question (25b) presupposes that there is at most one x, say a, that together with some y satisfies ψ. For example, *which man saw which women* presupposes that either no seeing took place whatsoever, or one man saw one or more women, and no other man saw any woman.

- Finally, the plural-singular question (25c) presupposes that there is at most one y, say b, that together with some x satisfies ψ. For example, *which men saw which woman* presupposes that either no seeing took place whatsoever, or one or more men saw one woman, and no other woman was seen by any any man.

3.3. **Pair-list questions and the absorption rule.** Higginbotham and May [2] noted that (7bi) is ambiguous in terms of its presupposition. One interpretation of (7bi), termed the *singular interpretation* in [2], presupposes that exactly one man saw exactly one woman, and no other seeing relationship holds. Another interpretation, termed the *bijective interpretation* or more commonly the *pair-list reading*, presupposes that no man saw more than one woman, and no woman saw more than one man, but does not presuppose that only one seeing relationship holds. The meaning (26a) is the singular interpretation; the bijective interpretation would be

(27) $$\{\, S \subseteq \mathrm{Dom}(x,y) \mid a_1 = a_2 \Leftrightarrow b_1 = b_2 \;\forall (a_1,b_1),(a_2,b_2) \in S \,\}.$$

In order to explain the ambiguity of (25a) between the singular interpretation (26a) and the bijective interpretation (27), Higginbotham and May hypothesized a rule of *absorption*, by which two or more adjacent raised wh-variables can be combined into a larger unit at LF. The application of the absorption rule is optional. For example, if we choose to skip absorption in (25a), the singular interpretation results as before; on the other hand, if we choose to apply absorption, the LF representation becomes

(28) $$[\{\mathbb{S}x, \mathbb{S}y\}]\, \psi(x,y).$$

Note that the ordering between x and y above is only notational, as indicated by the braces.

We now look at how the meaning (27) is assigned to (28). Consider in general a multiple-wh question with m singular wh-variables and l plural wh-variables absorbed into a larger unit:

(29) $$[\{\mathbb{S}x_1, \dots, \mathbb{S}x_m, \mathbb{P}y_1, \dots, \mathbb{P}y_l\}]\, \psi(x_1, \dots, x_m, y_1, \dots, y_l).$$ [4]

Higginbotham and May assign to (29) the meaning

(30) $$\begin{aligned}\big\{\, S \subseteq \mathrm{Dom}(&x_1, \dots, x_m, y_1, \dots, y_l) \\ &\mid \#\{\, a_k \mid (a_1, \dots, a_m, b_1, \dots, b_l) \in S \,\} = 1 \\ &\quad \forall k \in \{1, \dots, m\}, \forall (a_1, \dots, a_{k-1}, a_{k+1}, \dots, a_m, b_1, \dots, b_l) \\ &\qquad\qquad \in \mathrm{Dom}(x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_m, y_1, \dots, y_l) \,\big\}.\end{aligned}$$

In other words, (29) presupposes that each x_k has exactly one ψ-satisfying value given any set of values for all other variables.

Since (28) is the special case of (29) where $m = 2$ and $l = 0$, we can now compute the meaning of (28) by letting $m = 2$ and $l = 0$ in (30). Unfortunately, the result is *not* the bijective interpretation as formulated in (27). The mismatch is that (27) allows some men to not see any woman and some women to not be seen by any

[4] It was not explained in [2] how meanings are assigned to the result of 'partial absorption' in triple-wh questions. In other words, it is unclear how absorbed and un-absorbed wh-variables interact at LF. Therefore, we present here the semantics of not an absorbed wh-variable but an entire question with an absorbed wh-variable. Hence the lack of lambda-abstraction in (30).

man, while (30) requires each man to see exactly one woman and each woman to be seen by exactly one man. Higginbotham and May appear to not have noticed this mismatch.[5]

Not only is the absorption rule problematic in the singular-singular case (25a), it generates empirically unobserved meanings in the singular-plural case (25b) and the plural-singular case (25c). To see this, let $m = l = 1$ in (30) to compute the meaning of $[\{\mathbb{S}x, \mathbb{P}y\}]\,\psi(x, y)$. It turns out to be the set of all functions from $\mathrm{Dom}(y)$ to $\mathrm{Dom}(x)$. I was unable to come up with a double-wh question with one singular-which variable x and one plural-which variable y presupposing that, for each y, exactly one x satisfies the predicate.

4. Observing multiple-wh questions

To facilitate further analysis of the semantics of direct wh-questions, I now try to characterize the various presuppositions that direct multiple-wh questions can have. Again let V be the set of wh-variables involved in some direct wh-question q. It appears to me that what is presupposed by q is always expressible as the conjunction of zero or more statements of the form

(31) Given any set of values for (some variables), (at least one/at most one/ every) value for (some other variable) is such that values can be assigned to (all other variables) to satisfy ψ.

Note that, despite the numerous placeholders in its statement, (31) severely constrains the presuppositions that can be expressed in direct wh-questions. To start with, there is no way in this schema to express presuppositions such as that at least two men saw a woman (though it is easy to add such expressive power by augmenting the three quantifiers enumerated above). Furthermore, there is no way to express presuppositions such as that every teacher who gave some book to some student gave a book to every student. Nevertheless, all the presuppositions we have encountered so far in this paper are expressible.

4.1. **A notation for presuppositions.** Formally, let x be a variable in V and W be a subset of $V - \{x\}$. Let $a \in \mathrm{Dom}(x)$ be a value for the variable x, and $b \in \mathrm{Dom}(W)$ be a tuple of values for the variables W. If S is a theory (over V), then we say that (a, b) *can satisfy* S if there exists a tuple of values for the variables $V - W - \{x\}$, say $c \in \mathrm{Dom}(V - W - \{x\})$, such that (a, b, c) is an element of S. If for every value tuple b there is at least one a such that (a, b) can satisfy S, then we say that *at least one x can satisfy S given W*. If for every value tuple b there is at most one a such that (a, b) can satisfy S, then we say that *at most one x can satisfy S given W*. If for every value tuple b and every a such that (a, b) can satisfy S, then we say that *every x can satisfy S given W*. Now define the theory sets (i.e.,

[5] On [2, p. 45], it was stated that (7bi) "presupposes that the seeing relation is a one-to-one correspondence between a subset of the men and a subset of the women." Later, on p. 48, it was stated that each k in (30) "expresses the presupposition that the matrix predicate ψ is a function with respect to the k-th place." The wording "subset" is clearly inconsistent with the wording "function." Perhaps by "function" they meant "partial function," but then the bijective interpretation would no longer presuppose that any seeing took place.

presuppositions) 'at least one,' 'at most one,' and 'every' as follows:

(32) $\langle \exists x \mid W \rangle = \{\, S \subseteq \mathrm{Dom}(V) \mid \text{at least one } x \text{ can satisfy } S \text{ given } W \,\},$

(33) $\langle !x \mid W \rangle = \{\, S \subseteq \mathrm{Dom}(V) \mid \text{at most one } x \text{ can satisfy } S \text{ given } W \,\},$

(34) $\langle \forall x \mid W \rangle = \{\, S \subseteq \mathrm{Dom}(V) \mid \text{every } x \text{ can satisfy } S \text{ given } W \,\}.$

As stated above, the conjunctive closure of these primitives appear to me to be rich enough to cover presuppositions in direct wh-questions. For convenience, let us also introduce shorthand notation for the presuppositions 'exactly one' and 'no presupposition,' both of which can be derived from the primitives above by conjunction:

(35) $\langle \exists! x \mid W \rangle = \langle \exists x \mid W \rangle \cap \langle !x \mid W \rangle,$

(36) $\langle ? \rangle = \{\, S \subseteq \mathrm{Dom}(V) \,\} = 2^{\mathrm{Dom}(V)}.$

We now have a concise notation with which to discuss and manipulate presuppositions. To illustrate, let us use our new notation to summarize the presuppositions encountered in §3 above:

(16′) $[\![[\mathbb{P}x]\, \psi(x)]\!] = \langle ? \rangle.$

(21′) $[\![\{ [\mathbb{P}x], [\mathbb{P}y] \}\, \psi(x,y)]\!] = \langle ? \rangle.$

(23′) $[\![[\mathbb{S}x]\, \psi(x)]\!] = \langle \exists! x \rangle.$

(26′) a. $[\![\{ [\mathbb{S}x], [\mathbb{S}y] \}\, \psi(x,y)]\!] = \langle \exists! x \rangle \cap \langle \exists! y \rangle.$
b. $[\![\{ [\mathbb{S}x], [\mathbb{P}y] \}\, \psi(x,y)]\!] = \langle !x \rangle.$
c. $[\![\{ [\mathbb{P}x], [\mathbb{S}y] \}\, \psi(x,y)]\!] = \langle !y \rangle.$

(27′) $[\![[\{ \mathbb{S}x, \mathbb{S}y \}]\, \psi(x,y)]\!] = \langle !x \mid y \rangle \cap \langle !y \mid x \rangle \cap (\langle \exists x \rangle = \langle \exists y \rangle).$

(30′) $$[\![[\{ \mathbb{S}x_1, \dots, \mathbb{S}x_m, \mathbb{P}y_1, \dots, \mathbb{P}y_l \}]\, \psi(x_1, \dots, x_m, y_1, \dots, y_l)]\!] = \bigcap_{k=1}^{m} \langle \exists! x_k \mid x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_m, y_1, \dots, y_l \rangle.$$

From (30′) we have

(37) a. $[\![[\{ \mathbb{S}x, \mathbb{S}y \}]\, \psi(x,y)]\!] = \langle \exists! x \mid y \rangle \cap \langle \exists! y \mid x \rangle$, contradicting (27′).
b. $[\![[\{ \mathbb{S}x, \mathbb{P}y \}]\, \psi(x,y)]\!] = \langle \exists! x \mid y \rangle$, which is not empirically observed.

4.2. **Empirical observations.** For the moment, let us restrict ourselves to double-wh questions. If we categorize them by whether each wh-phrase is singular or plural, and take into account the surface (chain-head; linear) ordering between the two wh-phrases, then there are four kinds of double-wh questions: singular-singular, singular-plural, plural-singular, and plural-plural.[6] What can each category of double-wh questions presuppose?

It is instructive to solicit acceptability judgments on the questions themselves, placed in natural discourse settings. We should keep in mind, however, that simply that a question can be asked when the speaker has some expectations regarding the answer does not mean that these expectations are presupposed by the question. For example, the question *which men saw Mary* in (4b) does not presuppose that at least 25 men saw Mary. I have tried to use tests such as the ones described at the beginning of this paper to identify what is actually presupposed by each question.

[6]The deep (chain-tail) ordering between the two wh-phrases may also be relevant. To keep things simple, I have for the most part avoided reorderring constructions, such as passives.

Another complication is that native speakers of English often disagree on which multiple-wh questions make which presuppositions. The proposal in [2] does not account for this variation in the readings available to each native speaker. Indeed, even the same speaker sometimes gives different presupposition judgments at different times; perhaps this can be attributed to differences in stress pattern and imagined discourse context.

5. Singular wh-variables

As was the case in [2], the presuppositions that singular-singular wh-questions can make are most diverse and informative.

5.1. A variety of presupposition types. As one might expect, all speakers accept the reading that presupposes a unique pair satisfying the predicate:

(38) For the first time in Harvard history, a professor flunked a student!—Really? Which professor flunked which student?

a. I hear Harvard's registrar computer system crashed yesterday. It had trouble processing a flunking case; apparently no one has ever been flunked out of a class before.—Wow! Which professor flunked which student?—Well, actually, multiple professors were involved, and they flunked quite a few students. I'm not sure who exactly.

b. I hear Harvard's registrar computer system crashed yesterday. It had trouble processing a flunking case; apparently no one has ever been flunked out of a class before.—Wow! Which professors flunked which students?—*Well, actually, multiple professors were involved, and they flunked quite a few students. I'm not sure who exactly.

This reading is the "singular interpretation" in [2]; in the special case of double-wh questions, we term it the *unique pair presupposition.*

(39) Unique pair presupposition: $\langle \exists ! x\rangle \cap \langle \exists ! y\rangle$.

Note that the unique pair presupposition $\langle \exists ! x\rangle \cap \langle \exists ! y\rangle$ can also be written in two other ways:

(40) $\langle \exists ! x\rangle \cap \langle \exists ! y\rangle = \langle \exists ! x\rangle \cap \langle \exists ! y \mid x\rangle = \langle \exists ! y\rangle \cap \langle \exists ! x \mid y\rangle$.

In words, the presupposition that exactly one x can satisfy ψ and exactly one y can satisfy ψ is equivalent to the presupposition that exactly one x can satisfy ψ and exactly one y can satisfy ψ given x, and also equivalent to the presupposition that exactly one y can satisfy ψ and exactly one x can satisfy ψ given y.

In addition to the unique pair reading, each speaker also accepts another reading that does not presuppose that only one pair (a, b) satisfies $\psi(a, b)$. Depending on the speaker, the sentence and the discourse context, the alternative presupposition can be any one of the following:

(41)
a. Function presupposition: $\langle \exists ! y \mid x\rangle = \langle ! y \mid x\rangle \cap \langle \forall x\rangle$.
b. Partial function presupposition: $\langle ! y \mid x\rangle$.
c. Reverse function presupposition: $\langle \exists ! x \mid y\rangle = \langle ! x \mid y\rangle \cap \langle \forall y\rangle$.
d. Partial reverse function presupposition: $\langle ! x \mid y\rangle$.
e. Bijection presupposition: $\langle \exists ! y \mid x\rangle \cap \langle \exists ! x \mid y\rangle$.
f. Partial bijection presupposition: $\langle ! y \mid x\rangle \cap \langle ! x \mid y\rangle$.

An example of the function and partial function presuppositions is (42).

(42) In order to measure how well our new curriculum functions in the classroom, we had every student take all three tests, spaced throughout the academic year. As it turned out, every student passed some test.—Sounds good. Just out of curiosity, which student passed which test?—Well, actually, most students passed more than one test. Here is a detailed list.

We can distinguish between speakers who prefer the function presupposition and speakers who prefer the partial function presupposition with tests such as (43).

(43) In order to give our students additional opportunities to measure how much they have learned, we prepared three standardized tests, which they had the option to take but were not required to. In the end, no student took more than one test.—Just out of curiosity, which student took which test?—%Well, actually, some students didn't take any test at all. In any case, here is a detailed list.

The reverse function and partial reverse function presupposition are less often observed, a trend which we will try to explain below. An example is (44).

(44) Wow, that was a really great dinner party. Your cooking was excellent! The way people were grabbing seconds, it seems that every guest was impressed.—Oh, really? I'm glad people didn't all hate the food; I was a bit worried. Which dish impressed which guest?—Well, actually, some guests were impressed with several dishes. Don't be so humble.

5.2. **Functional effects on presuppositions.** Generally speaking, for a singular-singular question to make a function or partial function presupposition, it helps for the first variable x to be discourse-linked, animate (human), and/or logophoric. Similarly, for a singular-singular question to make a reverse function or partial reverse function presupposition, it helps for the second variable y to be discourse-linked, animate (human), and/or logophoric. The judgments are subtle, but I try to show some representative examples in (45–49). In each pair of sentences, the first one appears to be more often acceptable with the function or partial function presupposition than the second one, and the second one appears to be more often acceptable with the reverse function or partial reverse function presupposition than the first one.

(45) Discourse-linking effect:
- a. Which remark you made agitated which politician?
- b. Which remark agitated which politician you interviewed?

(46) Animacy effect in x position:
- a. Which counselor won over which kid?
- b. Which dessert won over which kid?

(47) Animacy effect in y position:
- a. Which kid likes which dessert?
- b. Which kid likes which counselor?

(48) Logophoricity effect between first person and third person:
- a. Which kid among us likes which counselor among them?
- b. Which kid among them likes which counselor among us?

(49) Logophoricity effect between second person and third person:

a. Which kid among you likes which counselor among them?
b. Which kid among them likes which counselor among you?

The effects of discourse-linking, animacy and logophoricity observed here are particularly relevant in establishing the reverse function and partial reverse function presuppositions. These effects are analogous to effects previous observed in quantifier scoping [3], suggesting a connection that I will make concrete shortly.

5.3. **An account of the observed meanings.** That there is any reading of a singular-singular question whose presupposition is not symmetric with respect to the two wh-variables is problematic for Higginbotham and May's absorption proposal, because if—as they theorized—the only way for a singular-singular question to presuppose a unique pair is through absorption, and absorption ignores any ordering between the two singular wh-variables, then absorption cannot possibly generate any non-symmetric reading.[7]

An alternative to the absorption proposal that, in my opinion, naturally emerges from the discussion so far is as follows. Suppose that the LF representation of double-wh questions does order the two wh-variables; in other words, let us remove the braces in (8b). In the singular-singular question

(50) $[\mathbb{S}x]\,[\mathbb{S}y]\,\psi(x,y),$

assign the meaning $\top$ to $\psi(x,y)$ as before. Also as before, assign the meaning (24) to $[\mathbb{S}x]$, and the analogous meaning to $[\mathbb{S}y]$. To generate the non-unique-pair presuppositions, we hypothesize that the first variable $[\mathbb{S}x]$ is ambiguous between the meaning (24) and another meaning, which for speakers who prefer function and reverse function presuppositions over partial function and partial reverse function presuppositions is

(51) $\lambda R.\bigl\{\, \bigcup_{a\in \mathrm{Dom}(x)} \{(a)\} \times f(a) \mid f\colon \mathrm{Dom}(x) \to R \,\bigr\}$

and for speakers who prefer the opposite is

(52) $\lambda R.\bigl\{\, \bigcup_{a\in D} \{(a)\} \times f(a) \mid D \subseteq \mathrm{Dom}(x),\ f\colon D \to R \,\bigr\}.$

Both (51) and (52) are functions that take as input a set R. The input R is the meaning of the partial question $[\mathbb{S}y]\,\psi(x,y)$. More specifically, R is a set of relations over the single variable y, i.e., a set of subsets of $\mathrm{Dom}(y)$ (in this case, the set of singleton subsets of $\mathrm{Dom}(y)$). What (51) does is compute the set of all theories $B \subseteq \mathrm{Dom}(x,y)$ with the property that each x-slice of B is an element of R.

Intuitively, to answer a function-presupposing singular-singular question is to specify, for each a in the domain of x, the corresponding b in the domain of y such that $\psi(a,b)$ is true. The singular wh-variable $[\mathbb{S}x]$ here is roughly equivalent to the quantifier-determiner *each*; it suggests to the listener that the variable x should be used as the "sorting key" in an answer to the question [3]. The semantics specified in (51) captures this intuition; the upshot is that the function (51) applied to R computes the set of functions from $\mathrm{Dom}(x)$ to $\mathrm{Dom}(y)$. Meanwhile, what (52) does is compute the set of *partial* functions from $\mathrm{Dom}(x)$ to $\mathrm{Dom}(y)$.

In sum, the ambiguity of $[\mathbb{S}x]$ between the three meanings (24), (51) and (52) give rise to the presuppositions (39), (41a) and (41b), respectively. Making the usual

---

[7]Formally, we say that a theory set $R \subseteq 2^{\mathrm{Dom}(x,y)}$ is *symmetric* (with respect to x and y) if and only if, for every theory $S \subseteq \mathrm{Dom}(x,y)$, the theory S and the theory $\{\,(a,b) \mid (b,a) \in S\,\}$ are either both in R or both outside of R.

assumption that wh-variables are quantifiers, all we need to generate the reverse presuppositions (41c) and (41d) is any account of quantifier scope inversion and quantifier scope ambiguity.

Before moving on to the two remaining presupposition types (41e) and (41f), let us address an issue with my proposed account: Why is the first wh-variable $[\mathbb{S}x]$ ambiguous and the second one $[\mathbb{S}y]$ unambiguous? In other words, why can $[\mathbb{S}y]$ not take on a meaning analogous to (51), namely

(53) $\lambda R.\big\{ \bigcup_{a\in \mathrm{Dom}(y)} \{(a)\} \times f(a) \mid f\colon \mathrm{Dom}(y) \to R \big\}$,

or a meaning analogous to (52), namely

(54) $\lambda R.\big\{ \bigcup_{a\in D} \{(a)\} \times f(a) \mid D \subseteq \mathrm{Dom}(x),\ f\colon D \to R \big\}$?

Let us consider (53) first. Suppose $[\mathbb{S}y]$ were to mean (53). Since $[\mathbb{S}y]$ is the last wh-variable in (50), the input R to this function is $\top$. It is easy to compute that the output of this function is a singleton, $\{\mathrm{Dom}(y)\}$. In other words, if $[\mathbb{S}y]$ were to mean (53), then $[\mathbb{S}y]\,\psi(x,y)$ would mean $\{\mathrm{Dom}(y)\}$. In other words, it would presuppose that every y satisfies the predicate (given any x). Under such a singleton presupposition, an interrogative involving the wh-variable y would not be felicitous—because the speaker has already presupposed the answer to the question. We thus conclude generally that a felicity constraint against singleton presuppositions in interrogatives naturally rules out the meaning (51) for the 'innermost' wh-variable in any multiple-wh question.

What about the 'partial' meaning (54)? An similar argument exists that it should be ruled out, but it is a bit weaker, as follows. Again, the input R is $\top$. Applying (54) to $\top$, we compute that the output is the power set $2^{\mathrm{Dom}(y)}$. In other words, if $[\mathbb{S}y]$ were to mean (54), then $[\mathbb{S}y]\,\psi(x,y)$ would mean $2^{\mathrm{Dom}(y)}$. In other words, it would make no presupposition whatsoever regarding the set of y values satisfying the predicate (even given x). Under such a null presupposition, the singular wh-variable $[\mathbb{S}y]$ may as well be replaced by a plural wh-variable $[\mathbb{P}y]$. Just as in the previous paragraph, this argument generalizes to an argument ruling out the meaning (52) for the 'innermost' wh-variable in any multiple-wh question.

5.4. **The bijective interpretation.** On the topic of singular-singular questions, it remains for us to explain the bijection presupposition (41e) and the partial bijection presupposition (41f), which together corresponds to the bijective interpretation described in [2]. A possible explanation for these symmetric presuppositions is to appeal to a general discourse principle:

(55) Unambiguous sentences are preferred over ambiguous ones.

While vague, this principle motivates a slightly more concrete rule regarding presuppositions:

(56) Presuppose that each utterance is unambiguous: In possible world semantics, presuppose that the real world is one on which the multiple possible meanings of each utterance agree, in terms of truth-conditional or other semantic content.

A simple example of the kind of linguistic phenomena this rule might help explain is the sentence

(57) I don't smoke anymore.

Suppose that we consider (57) ambiguous between the following meanings:

(58) a. I quit smoking.
b. I do not smoke.

In possible worlds where I never even started smoking, the two meanings disagree on truth-conditional content. Hence, the explanation goes, cooperative speakers and listeners consider (57) to presuppose that I have smoked in the past.[8]

In (56) above, I make reference to semantic content other than truth-conditional content because—as one would expect—I suggest applying the rule to interrogatives as well as declarative sentences. Given that presupposition effects in singular-singular questions are as subtle as described above, many singular-singular questions are ambiguous in scope (i.e., ambiguous in terms of which wh-variable is ordered before the other at LF). To eliminate this ambiguity, we—or rather, cooperative and cautious speakers and listeners of singular-singular questions—take the intersection (conjunction) of the function presupposition and the reverse function presupposition, or of the partial function presupposition and the partial reverse function presupposition. The former intersection is equal to the bijection presupposition; the latter is equal to the partial bijection presupposition.

If this explanation is on the right track, we would expect singular-singular questions that are more ambiguous according to usual quantifier scoping generalizations to more likely receive a bijective interpretation. Consider (59a) and (59b), the two sentences extensively discussed during the analysis of interrogatives in [2].

(59) a. Which man saw which woman? (1.11 in [2])
b. Which student got which grade (on the exam)? (1.41 in [2])

According to [2], (59a) is ambiguous between the unique pair presupposition and the partial bijection presupposition. Also according to [2], a reading exists for (59b) that makes only the partial function presupposition. In other words, the partial reverse function presupposition may be canceled in (59b). Higginbotham and May explains this cancellation by noting that the word *grade* is "ambiguous as between the abstract letters 'A,' 'B,' etc. and their applications to [each student]."

We do not deny that this explanation is plausible, but our line of thinking above suggests an additional consideration: Note that men, women and students are all animate while grades and applications thereof are not. Therefore, we would expect (59a) to be more ambiguous in scope than (59b) is. Consequently, (59a) is more likely to be read as making the partial reverse function presupposition than (59b) is. One advantage of this additional consideration is that it also explains why the native speakers I consulted did not find a partial reverse function presupposition in the sentence *which student attended which party.*

A final comment on the bijective interpretation: It appears that the native speakers I consulted seldom or never read singular-singular questions as making the bijection presupposition or the partial bijection presupposition. This may be because the test sentences I used are different from those used by Higginbotham and May.

[8]To hand-wave even further: Perhaps (58a) corresponds to the bracketing

(i) I [don't smoke] anymore.

and (58b) corresponds to the bracketing

(ii) I don't [smoke anymore].

Nevertheless, the bijective interpretation does seem to be implicated in the following phenomenon: In most contexts where a singular-singular question is acceptable, so is a plural-plural question. For example, in (42), the question *which students passed which tests* would be just as acceptable as *which student passed which test*. As far as I can tell, there are only two contexts where a singular-singular question is acceptable and a plural-plural question is not. The first context is if the speaker knows that only one pair satisfies the predicate. In this case, a singular-singular question (taking the unique pair reading) would be acceptable, while a plural-plural question would be unacceptable:

(60) a. Which presidential candidate won the election last year with which vice presidential candidate?
b. *Which presidential candidates won the election last year with which vice presidential candidates?

The second context is if the speaker knows that the predicate is a permutation, in other words a bijection between $\mathrm{Dom}(x)$ and $\mathrm{Dom}(y)$. In this case, a singular-singular question (taking the bijection reading, or some other reading that is not the unique pair reading) would be acceptable, while a plural-plural question would be unacceptable:

(61) a. Which peg goes in which hole?
b. *Which pegs go in which holes?

Note that these two contexts are precisely the two contexts in which scope ambiguity would not affect the semantic content of a singular-singular question. Again a very tentative suggestion can be made—namely that potential ambiguity in a sentence affects its acceptability.

6. Plural wh-variables

Presupposition types in singular-plural, plural-singular and plural-plural questions are much simpler than those in singular-singular questions. The following types of presupposition are available in singular-plural questions:

(62) a. Unique first variable presupposition: $\langle \exists! x \rangle$.
b. Multi-function presupposition: $\langle \forall x \rangle$.
c. No presupposition: $\langle ? \rangle$. (May not be available.)

An example of the unique first variable presupposition, the most available as well as the most expected to be available, is the following.

(63) For the first time in Harvard history, a professor flunked multiple students in a single class!—Really? Which professor flunked which students?

This presupposition type is easily explained in our account. Begin by noting that there are two possible quantifier scopings—one with the singular wh-variable scoping over the plural wh-variable:

(64) $[\mathbb{S}x]\,[\mathbb{P}y]\,\psi(x,y)$,

and one with the plural wh-variable (inverse-) scoping over the singular wh-variable:

(65) $[\mathbb{P}y]\,[\mathbb{S}x]\,\psi(x,y)$.

In either possible LF representation above, take $[\mathbb{S}x]$ to mean (24) and $[\mathbb{P}y]$ to mean (20). The requisite presupposition then falls out of a simple set-theoretic computation analogous to (26b).

The multi-function presupposition, and perhaps the null presupposition for some speakers, are shown below.

(66) Each student in the class was assigned to be graded by one TF.—But this way, some TFs must have graded more harshly than others. Let's see... Which TF graded which students?

The multi-function presupposition is generated in our analysis by taking $[\mathbb{S}x]$ to mean (51) and $[\mathbb{P}y]$ to mean (20) in (64). The null presupposition can be thought of as the partial multi-function presupposition, and is generated in our analysis by taking $[\mathbb{S}x]$ to mean (52) and $[\mathbb{P}y]$ to mean (20) in (64). The null presupposition is the least available of all three presupposition types in (62), if it is available at all. Presumably, this is because the same presupposition is expressed unambiguously by a plural-plural question.

Note that in the inverse-scope LF representation (65), the only meaning available for $[\mathbb{S}x]$ is (24)—the meanings (51) and (52) are not available, for reasons explained at the end of §5.3.

Turning to plural-singular questions, the following types of presuppositions are available:

(67) a. Unique second variable presupposition: $\langle \exists! y \rangle$.
b. Reverse multi-function presupposition: $\langle \forall y \rangle$.
c. No presupposition: $\langle ? \rangle$. (May not be available.)

The unique second variable presupposition is by far the reading most often found natural:

(68) How did the students do on the exam?—Well, mostly everyone did fine, but there was one problem that many students had trouble with.—Hrm. Which students had trouble with which problem?

The reverse multi-function presupposition, and perhaps the null presupposition for some speakers, are illustrated below.

(69) Wow, that was a really great dinner party. Your cooking was excellent! The way people were grabbing seconds, it seems that every guest was impressed.—Oh, really? I'm glad people were happy with the food; I was a bit worried. Which dishes impressed which guest, anyway?

We can account for the plural-singular presupposition types (67) in our framework just as we did for singular-plural presupposition types (62), except what used to be linear-scope at LF is now inverse-scope, and vice versa. The reverse multi-function presupposition is very difficult to elicit in plural-singular questions, even compared to the multi-function presupposition in singular-plural questions. This difficulty is explained by the fact that scope inversion is required for the reverse multi-function presupposition. Making the second wh-variable an animate object helps, as it did in (47).

Finally we come to plural-plural questions. Our account of them is no different from that of [2] as summarized in §3.1.

References

[1] Higginbotham, J. (1996) The semantics of questions. In S. Lappin (ed.), *The Handbook of Contemporary Semantic Theory*, 361–383. Oxford: Blackwell.

[2] Higginbotham, J. and May, R. (1981) Questions, quantifiers, and crossing. *Linguistic Review*, 1, 41–79.

[3] Kuno, S. (1991) Remarks on quantifier scope. In H. Nakajima (ed.), *Current English Linguistics in Japan*, 261–287. Berlin/New York: Mouton de Gruyter.

[4] Levi, I. (1967) *Gambling with Truth*. New York: Alfred A. Knopf.

Harvard University, 33 Oxford Street, Cambridge, MA 02138, USA
E-mail address: `ken@digitas.harvard.edu`